

# Exam I, Math 222, section 7

September 22, 2008

**Problem 1.** a) Let  $f(x) = x + 2e^x$ .

i) Prove that  $f$  has an inverse function  $g$  and find the domain of  $g$  (**5 points**).

ii) Compute  $f(0)$ ,  $g(2)$  and  $g'(2)$  (**5 points**).

iii) What is  $f(g(f(g(7))))$ ? (**3 points**)

b) (**5 points**) Find the inverse function of  $f(x) = e^{\sqrt{x}}$ ,  $x \in (0, \infty)$ .

**Problem 2.** a) (**5 points**) Solve the equation

$$\log_3(x+2) + \log_3(4-x) = 2$$

b) (**5 points**) Find the derivative of each of the functions

$$f(x) = x^{\arctan x} \qquad g(x) = \log_{e^x} 2$$

**Problem 3.** Compute the following limits (**4 points each**). If you use L'Hospital's rule, show where you use it and explain what type of limit you are using it on.

$$\text{a) } \lim_{x \rightarrow \infty} (e^x + 3)^{1/(x+1)} \qquad \text{b) } \lim_{x \rightarrow 0} \frac{x^2 - x}{\cos x} \qquad \text{c) } \lim_{x \rightarrow 0} \frac{\sin x - x}{x^3} \qquad \text{d) } \lim_{x \rightarrow \infty} x \left( \frac{\pi}{2} - \arctan x \right)$$

**Problem 4.** Compute the following integrals (**5 points each**):

$$\text{a) } \int_1^e \frac{\sqrt{\ln x}}{x} dx \qquad \text{b) } \int_0^{1/\sqrt{2}} \frac{2x dx}{\sqrt{1-x^4}} \qquad \text{c) } \int \frac{4e^{2x} dx}{1+4e^{4x}}$$

**Problem 5.** A bacteria culture starts with 1500 bacteria and grows with the growth rate proportional to the number of bacteria. After 3 hours the population is 12000.

1) Find an expression for the number of bacteria after  $t$  hours. **6 points**

2) How long does it take for the number of bacteria to double? **5 points**