Exam III, Math 222

December 3, 2008

Problem 1. a) Consider the parametric curve $x = t^2 + t + 1$, $y = 4t^3 + 3t^2 + 2$. Find $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ as functions of t. Find the equation of the line tangent to the curve at the point $(1, 1)$. (**5 points**)

b) Sketch the curve given in polar coordinates by the equation $r = \theta^2$, $\theta \in [0, 2\pi]$. Find a parametric equation of this curve in Cartesian coordinates (use θ as the parameter). Compute the length of this curve. (**5 points**)

c) The curve $x = t^2 + 2$, $y = \frac{1}{3}t^3 - t + 2$, $t \in [0, 1]$ is revolved about the y-axis. Compute the area of the resulting surface. (**5 points**)

Problem 2. Compute the following infinite sums (**3 points each**):

$$\text{a) } \sum_{n=2}^{\infty} \left(\frac{-2}{3}\right)^{n-1} \qquad \text{b) } \sum_{n=1}^{\infty} \left(\frac{1}{ne^n} - \frac{1}{(n+1)e^{n+1}}\right) \qquad \text{c) } \sum_{n=1}^{\infty} \frac{(1-\frac{1}{e})^n}{n}$$

Problem 3. Determine whether the following series is absolutely convergent, conditionally convergent or divergent. Explain what test you are applying and verify all the conditions necessary to apply the test. (**5 points each**)

$$\text{a) } \sum_{n=1}^{\infty} (-1)^n \cos\left(\frac{1}{n}\right) \quad \text{b) } \sum_{n=1}^{\infty} \frac{1}{\sqrt[3]{n^4 - n^2 + 1}} \quad \text{c) } \sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n\sqrt{\ln n + 1}} \quad \text{d) } \sum_{n=1}^{\infty} \left(\frac{1}{2n} - 1\right)^{2n^2}$$

Problem 4. a) Find the radius and interval of convergence of the power series $\sum_{n=1}^{\infty} \frac{2n+1}{3n^2+2}(x-1)^n$. Carefully justify your answer. (**8 points**)

b) Give the power series expansion centered at 0 for the following functions and state radius of convergence (**3 points each**):

$$\text{i) } \sqrt{1 - 2x^2} \qquad \text{ii) } \frac{d}{dx}\left(\frac{1}{1-x}\right) \qquad \text{iii) } \int \cos(\sqrt{x})dx$$

c) Use i) of part b) to compute the 6th derivative of $\sqrt{1 - 2x^2}$ at 0. (**3 points**)

Problem 5. a) Find the Taylor series for $f(x) = \sin x$ centered at $\pi/2$. Use Taylor's inequality to prove that the Taylor series converges to $f(x)$ for all x. (**6 points**)

b) Find the degree 3 Taylor polynomial centered at $\pi/4$ for $f(x) = \ln \cos x$. (**5 points**)