SERIES GUIDE

Answers, not solutions, are given to the "supplementary problems" in the book. We will want to see **solutions** on Test 3 and the Final – – in sufficient detail that we believe you know what you are doing.

In this note we use "a_n" for the *terms* of a series, and S_n for the *partial sums*. Do not use s_n and S_n for these, as the book does, since it is difficult for us to to distinguish upper and lower case "s" on a test.

BE CAREFUL WITH THE WORD "IT"! Do **not** write: "It converges." or "It diverges." There are too many potential "its". If you are considering the series $\sum a_k$, there is the sequence $\langle a_k \rangle$ of terms of the series and there is the sequence $\langle S_n \rangle$ of partial sums. If you are doing either a root test or a ratio test, you are working with another sequence. If you are doing some sort of comparison test, you are using at least two additional sequences.

You may use "$\to$" for "$\lim\limits_{n\to\infty}$"; for example, "$a_n \to L$" means "$\lim\limits_{n\to\infty} a_n = L$".

You should be familiar with the limits from the "useful limits" list, available on the Calc2 website, since you can use them without further justification. These limits are used several times below.

The following write-ups would give you full credit on an exam:

Example 1. Does $\displaystyle\sum_{n=1}^{\infty}(-1)^n \frac{n^2+2n}{n^2+7n+10^{10}}$ converge or diverge?

Soln. Let $a_n = (-1)^n \dfrac{n^2+2^n}{n^2+7n+10^{10}}$. $\lim\limits_{n\to\infty}|a_n| = \lim\limits_{n\to\infty}\dfrac{n^2}{n^2} = 1$ (from the "useful limits" list) so $a_n \not\to 0$ and therefore $\displaystyle\sum_{n=1}^{\infty} a_n$ diverges.

Example 2. Does $\displaystyle\sum_{n=1}^{\infty}(-1)^n \left(\frac{\pi+1}{5}\right)^n$ converge or diverge?

Soln.. $\pi < 4$ so $\pi + 1 < 5$ and $\dfrac{\pi+1}{5} < 1$. Thus $\left|-1\left(\dfrac{\pi+1}{5}\right)\right| < 1$ and therefore the series $\displaystyle\sum_{n=1}^{\infty}\left(-1\left(\frac{\pi+1}{5}\right)\right)^n$ is a convergent geometric series.

Example 3. Does $\sum_{n=3}^{\infty} \frac{4}{n^2-4}$ converge or diverge?

Soln. $\sum_{n=1}^{\infty} \frac{1}{n^2}$ is a convergent p-series with $p = 2 > 1$.

Limit Comparison Test: Note that the terms of both series are positive.

$$\frac{\frac{4}{n^2-4}}{\frac{1}{n^2}} = \frac{4n^2}{n^2-4} \to 4$$

(from the "useful limits" list). Then $0 \leq 4 < \infty$ so $\sum_{n=3}^{\infty} \frac{4}{n^2-4}$ converges by limit comparison with $\sum_{n=1}^{\infty} \frac{1}{n^2}$.

Example 4. Find the set of x's for which the series $\sum_{n=1}^{\infty} \frac{(2x-5)^n}{n}$ converges.

Soln. Let $a_n = \frac{(2x-5)^n}{n}$.

Root Test: $|a_n|^{1/n} = \frac{|2x-5|}{n^{1/n}} \to |2x-5| \begin{cases} < 1 & \text{conv} \\ = 1 & ? \\ > 1 & \text{div} \end{cases}$

($n^{\frac{1}{n}} \to 1$ is on the "useful limits" list).

$|2x-5| < 1 \iff -1 < 2x-5 < 1 \iff 4 < 2x < 6 \iff 2 < x < 3$

End Points:

When $x = 2$ the original series becomes $\sum_{n=1}^{\infty} (-1)^n \left(\frac{1}{n}\right)$ which converges by the Alternating Series Test since $\left\langle \frac{1}{n} \right\rangle$ is decreasing and $\frac{1}{n} \to 0$.

When $x = 3$ the original series becomes $\sum_{n=1}^{\infty} \frac{1}{n}$ which is a divergent p-series ($p = 1 \leq 1$) (or "which is the Harmonic Series – known to diverge").

Thus the original series converges if $\boxed{2 \leq x < 3}$.

Notes:

(A) Let us know what test you are using and how it applies.
(B) If you've already used the root or ratio test to show that the series converges for x in (a, b) and diverges for x outside of $[a, b]$, don't even try to use the root or ratio test on the endpoints. You've already shown it to be inconclusive there.