Exam I, Math 222, section 1

February 26, 2013

Problem 1. a) Let $f(x) = \arctan(\ln x)$. Find a formula for f^{-1}. (**5 points**)

b) Let
$$f(x) = \frac{\arctan x}{\frac{\pi}{2} + \arctan x}, \quad x \in (-\infty, \infty).$$
Prove that f has an inverse and find the domain of f^{-1}. (**6 points**)

c) The function $f(x) = x + 2e^x$, $x \in (-\infty, \infty)$ is one to one. Compute $(f^{-1})'(2)$. (**5 points**)

Problem 2. a) Compute $\cos\left(\arcsin\dfrac{12}{13}\right)$. (**5 points**)

b) Differentaite the following function: $f(x) = x^x \cdot 3^{x^2} \cdot 5^{\sin x}$. (**5 points**)

c) Solve the following equation: $\log_3(x+4) - \log_3(4-x) = 1$. (**5 points**)

Problem 3. Compute the following limits (**5 points each**). If you use L'Hospital's rule, show where you use it and explain what type of limit you are using it on.

a) $\displaystyle\lim_{x\to\infty} (e^x + 3)^{\frac{1}{x+1}}$ b) $\displaystyle\lim_{x\to 0} \frac{x^2 - x}{\cos x}$ c) $\displaystyle\lim_{x\to 0} \frac{\sin x - x}{x^3}$ d) $\displaystyle\lim_{x\to 0} \cot^2 x \cdot \ln\cos x$

Problem 4. Compute the following integrals (**6 points each**):

a) $\displaystyle\int_1^e \frac{\sqrt{\ln x}}{x}\,dx$ b) $\displaystyle\int e^{2x}\cos x\,dx$ c) $\displaystyle\int_1^{\sqrt{3}} \frac{dx}{x^2\sqrt{4-x^2}}$ d) $\displaystyle\int \tan^8 x \sec^4 x\,dx$.

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The following problems are optional. You may earn extra points (8 points each), but work on this problems only after you are done with the above problems.

Problem 5. Using integration by parts derive the following formula for $n \geq 2$:
$$\int \frac{f(x)}{(x-a)^n}\,dx = \frac{f(x)}{(1-n)(x-a)^{n-1}} + \frac{1}{n-1}\int \frac{f'(x)}{(x-a)^{n-1}}\,dx.$$

Problem 6. Suppose that $f(0) = 3$, $f(3) = 6$, $f'(3) = 4$ and $f''(x)$ is continuous. Calculate $\int_0^3 x f''(x)\,dx$.