# Practice Problems for Exam 1

**Problem 1.** Prove or disprove the formula.

$$\cosh(\ln(x)) - \sinh(\ln(x)) = \frac{1}{x}, \quad x > 0.$$

$$\tan(\arcsin(x)) = \frac{x}{\sqrt{1-x^2}}, \quad -1 < x < 1.$$

$$\arcsin(x) + \arccos(x) = \frac{\pi}{2}, \quad -1 \leq x \leq 1.$$

**Problem 2.** Consider the function $f(x) = \cosh(\sqrt{x})$.

1. Find the domain of $f$.
2. Find the inverse $f^{-1}$ of $f$.
3. Find the domain of $f^{-1}$.

Practice this problem with other functions.

(a) $f(x) = \sinh(\sqrt{x})$ (b) $f(x) = \dfrac{e^x}{1+e^x}$ (c) $f(x) = \exp(1/x)$ (d) $f(x) = \ln(1+e^x)$.

**Problem 3.** Is the function invertible? Justify your answer.

(a) $f(x) = x + e^x$ (b) $f(x) = x\sinh(x)$ (c) $f(x) = x^2\sinh(x)$

(d) $f(x) = x\cosh(x)$ (e) $f(x) = x^2\cosh(x)$ (f) $f(x) = \tan(\arcsin(x))$

**Problem 4.** The function $f(x) = x\cosh(x)$ is invertible. Find the derivative $(f^{-1})'(a)$ of its inverse $f^{-1}$ at $a = 1$. Practice this with other functions.

(a) $f(x) = x + x^3 \quad a = 2$ (b) $f(x) = e^{\sinh(x)} \quad a = 1$

(c) $f(x) = \sinh(x + x^3) \quad a = 0$ (d) $f(x) = \sinh(x) + \sinh^3(x) \quad a = 0$

(e) $f(x) = x^2\sinh(x) \quad a = \sinh(1)$ (f) $f(x) = \ln(1 + x^5) \quad a = \ln(2)$

**Problem 5.** Find the derivative.

(a) $\dfrac{d}{dx}\, 3^{2x}$ (b) $\dfrac{d}{dx}\, \log_3(7x)$ (c) $\dfrac{d}{dx}\, \log_3(\sqrt{1+x^2})$

(d) $\dfrac{d}{dx}\, e^{arcsin(x)}$ (e) $\dfrac{d}{dx}\, \ln(\pi + \arccos(x))$ (f) $\dfrac{d}{dx}\, \arccos(1 - e^x)$

(g) $\dfrac{d}{dx}\, 2^{\cosh(x)}$ (h) $\dfrac{d}{dx}\, \cosh(x)\sinh(x)$ (i) $\dfrac{d}{dx}\, \sin(\sinh(x))$

(j) $\dfrac{d}{dx}\, x\exp(x)$ (k) $\dfrac{d}{dx}\, \exp(\arctan(x))$ (l) $\dfrac{d}{dx}\, \arcsin(1 - x^2)$

(m) $\dfrac{d}{dx}\, \ln(1 + e^x)$ (n) $\dfrac{d}{dx}\, \arctan(\cosh(x))$ (o) $\dfrac{d}{dx}\, \arctan(x)\cosh(x)$

(p) $\dfrac{d}{dx}\, \cosh(x)^{\sinh(x)}$ (q) $\dfrac{d}{dx}\, (1 + cos^2(x))^{\sin(x)}$ (r) $\dfrac{d}{dx}\, \cosh(x)^{1+x^2}$

**Problem 6.** Find the derivative

(a) $\frac{d}{dx}\ln(\ln(x))$ (b) $\frac{d}{dx}\ln(\ln(1+x^2))$ (c) $\frac{d}{dx}\ln(\ln(1+e^x))$ (d) $\frac{d}{dx}\ln\left(\frac{\ln(1+x^2)}{1+x^2}\right)$

(e) $\frac{d}{dx}x^{\sqrt{x}}$ (f) $\frac{d}{dx}(x^x)^x$ (g) $\frac{d}{dx}x^{(x^x)}$ (h) $\frac{d}{dx}x^{(x^2)}$

**Problem 7.** Use logarithmic differentiation to find the derivative.

(a) $\frac{d}{dx}\frac{x^3(1+x^2)^5}{\sqrt{1-x^2}}$ (b) $\frac{d}{dx}\frac{e^x(1+x^2)^9}{\sqrt{1-x^2}}$ (c) $\frac{d}{dx}\frac{(x+xe^x)\sqrt{1+x^2}}{(2+\exp(x))^2}$

(d) $\frac{d}{dx}e^{x^2}\cosh(1+x^2)x^x$ (e) $\frac{d}{dx}\frac{\cosh(x)}{\cosh(\sqrt{x})(1+\sinh^2(x)}$ (f) $\frac{d}{dx}\frac{(1+x^2)^3}{e^x\cosh(x^2)}$

**Problem 8.** Evaluate the integral.

(a) $\int xe^{x^2}\,dx$ (b) $\int x3^{x^2}\,dx$ (c) $\int\frac{e^{1/x}}{x^2}\,dx$ (d) $\int\cos(x)7^{\sin(x)}\,dx$

(e) $\int\frac{\pi^{\arctan(x)}}{1+x^2}\,dx$ (f) $\int\frac{e^{\arcsin(x)}}{\sqrt{1-x^2}}\,dx$ (g) $\int e^x\sec^2(e^x)\,dx$ (h) $\int\frac{e^x\ln(1+e^x)}{1+e^x}\,dx$

(i) $\int e^{1+\ln(x)}\,dx$ (j) $\int\frac{2^{\tan(x)}}{\cos^2(x)}\,dx$ (k) $\int\frac{e^x}{\sqrt{1-e^{2x}}}\,dx$ (l) $\int\frac{e^x}{1+e^x}\,dx$

**Problem 9.** Evaluate the integral.

(a) $\int\cot(x)\ln(\sin(x))\,dx$ (b) $\int\frac{\ln(\tan(x))}{\sin(x)\cos(x)}\,dx$ (c) $\int\tan(x)\ln(\cos(x))\,dx$

(d) $\int\frac{1}{\arcsin(x)\sqrt{1-x^2}}\,dx$ (e) $\int\frac{\arctan(x)}{1+x^2}\,dx$ (f) $\int\frac{2x+1}{x^2+x+1}\,dx$

(g) $\int\frac{\ln(\sqrt{x})}{x}\,dx$ (h) $\int\frac{\ln(x^2)}{x}\,dx$ (i) $\int\frac{1}{x\ln(x)}\,dx$

(j) $\int\frac{1}{x\ln(x^2)}\,dx$ (k) $\int\frac{1}{\ln^2(x)x}\,dx$ (l) $\int\frac{1}{x\sqrt{\ln(x)}}\,dx$

(m) $\int\frac{1}{x+x\ln(x^2)}\,dx$ (n) $\int\frac{1}{x+x\ln(\sqrt{x})}\,dx$ (o) $\int\frac{x}{(1+x^2)\ln(1+x^2)}\,dx$

**Problem 10.** Evaluate the integral.

(a) $\displaystyle\int \frac{1}{1+x^2}\,dx$ (b) $\displaystyle\int \frac{x}{1+x^2}\,dx$ (c) $\displaystyle\int \frac{x^2}{1+x^2}\,dx$ (d) $\displaystyle\int \frac{x^2-1}{1+x^2}\,dx$

(e) $\displaystyle\int \frac{x^3}{1+x^2}\,dx$ (f) $\displaystyle\int \frac{x^4}{1+x^2}\,dx$ (g) $\displaystyle\int \frac{x^5}{1+x^2}\,dx$ (h) $\displaystyle\int \frac{x^6}{1+x^2}\,dx$

(i) $\displaystyle\int \frac{x}{1+x^4}\,dx$ (j) $\displaystyle\int \frac{x^3}{1+x^4}\,dx$ (k) $\displaystyle\int \frac{x}{(1+x^2)^2}\,dx$ (l) $\displaystyle\int \frac{x^3}{(1+x^2)^2}\,dx$

**Problem 11.** Evaluate the integral.

(a) $\displaystyle\int \frac{\sin(x)}{1+\cos^2(x)}\,dx$ (b) $\displaystyle\int \frac{\sin(x)\cos(x)}{1+\cos^2(x)}\,dx$ (c) $\displaystyle\int \frac{\sin(x)\cos^2(x)}{1+\cos^2(x)}\,dx$

(d) $\displaystyle\int \frac{\sinh(x)}{1+\cosh^2(x)}\,dx$ (e) $\displaystyle\int \frac{\sinh(x)\cosh(x)}{1+\cosh^2(x)}\,dx$ (f) $\displaystyle\int \frac{\sinh(x)\cosh^2(x)}{1+\cosh^2(x)}\,dx$

**Problem 12.** Evaluate then integral.

(a) $\displaystyle\int \frac{1}{\sqrt{1-x^2}}\,dx$ (b) $\displaystyle\int \frac{x}{\sqrt{1-x^2}}\,dx$ (c) $\displaystyle\int \frac{x^2}{\sqrt{1-x^2}}\,dx$ (d) $\displaystyle\int \frac{x^3}{\sqrt{1-x^2}}\,dx$

(e) $\displaystyle\int \frac{1}{\sqrt{9-4x^2}}\,dx$ (f) $\displaystyle\int \frac{x}{\sqrt{9-4x^2}}\,dx$ (g) $\displaystyle\int \frac{x^2}{\sqrt{9-4x^2}}\,dx$ (h) $\displaystyle\int \frac{x^3}{\sqrt{9-4x^2}}\,dx$

**Problem 13.** Evaluate then integral.

(a) $\displaystyle\int \frac{\sinh(x)}{1+\sinh^2(x)}\,dx$ (b) $\displaystyle\int \sinh(x)\cosh(x)\,dx$ (c) $\displaystyle\int \tanh(x)\,dx$

(d) $\displaystyle\int \frac{\cosh(x)}{1+\sinh^2(x)}\,dx$ (e) $\displaystyle\int (\cosh^2(x)+\sinh^2(x))\,dx$ (f) $\displaystyle\int \frac{\cosh(x)}{\sqrt{1-\sinh^2(x)}}\,dx$

(g) $\displaystyle\int \frac{\cosh(x)}{\cosh^2(x)-1}\,dx$ (h) $\displaystyle\int \frac{\cosh(x)}{\cosh^2(x)-\sinh^2(x)}\,dx$ (i) $\displaystyle\int \frac{e^{2x}-e^{-2x}}{e^{2x}+e^{-2x}}\,dx$

**Problem 14.** Find the limit without using L'Hospital's Rule.

(a) $\displaystyle\lim_{x\to 1}\frac{x^2-1}{x-1}$ (b) $\displaystyle\lim_{x\to 0}\frac{e^{2x}-1}{1-e^x}$ (c) $\displaystyle\lim_{x\to\infty}\sqrt{2+x}-\sqrt{1+x}$

(d) $\displaystyle\lim_{x\to\infty}\ln(1+x^2)$ (e) $\displaystyle\lim_{x\to\infty}\tanh(x)$ (f) $\displaystyle\lim_{x\to\infty}\sqrt{1+2x}-\sqrt{1+x}$

(g) $\displaystyle\lim_{x\to 0}\frac{\sinh(x)}{x}$ (h) $\displaystyle\lim_{x\to\infty}\ln(1+2x)-\ln(1+x)$ (i) $\displaystyle\lim_{x\to\infty}\ln(x^2)-\ln(1+x)$

**Problem 15.** Find the limit.

(a) $\lim_{x\to 0} \dfrac{\sinh(x)}{x}$

(b) $\lim_{x\to 0} \dfrac{\cosh(x)-1}{\sinh^2(x)}$

(c) $\lim_{x\to 0} \dfrac{x\sinh(x)}{\cosh(x)-1}$

(d) $\lim_{x\to 0^+} \sinh(x)\ln(x)$

(e) $\lim_{x\to 0^+} \sqrt{x}\ln(x)$

(f) $\lim_{x\to 0^+} x(\ln(x))^3$

(g) $\lim_{x\to 0^+} \sinh(x)\ln(\cosh(x))$

(h) $\lim_{x\to 0} \ln(\cosh(x))\cot(x)$

(i) $\lim_{x\to 0^+} \ln(1+x)\ln(x)$

(j) $\lim_{x\to \infty} \dfrac{\ln\ln x}{\ln(x)}$

(k) $\lim_{x\to \infty} \ln(x) - \ln(\sqrt{1+x^2})$

(l) $\lim_{x\to \infty} \dfrac{\ln^2 x}{x}$

(m) $\lim_{x\to 0} \dfrac{x}{\arcsin(x)}$

(n) $\lim_{x\to 0^+} \sqrt{x}^{\sqrt{x}}$

(o) $\lim_{x\to 0} \cosh(x)^x$

(p) $\lim_{x\to 0^+} (1+\sqrt{x})^{ln(x)}$

(q) $\lim_{x\to 0^+} (1+\dfrac{2}{\sqrt{x}})^{\sqrt{x}}$

(r) $\lim_{x\to 0^+} \cos(x)^{\ln(x)}$

(s) $\lim_{x\to \infty} \cosh(x) - \sinh(x)$

(t) $\lim_{x\to \infty} \mathbf{sech}(x) + \tanh(x)$

(u) $\lim_{x\to \infty} \dfrac{\cosh^2(x)+\sinh^2(x)}{2+\sinh^2(x)}$