Exam II, Math 222, section 1 April 30, 2013

Problem 1. Compute the following infinite sum $\sum_{n=1}^{\infty} \frac{(\ln 2)^n}{2^n n!}$. (5 points)

Problem 2. Determine whether the following series is absolutely convergent, conditionally convergent or divergent. Explain what test you are applying and verify all the conditions necessary to apply the test. (6 points each)

a) $\sum_{n=1}^{\infty}(-1)^n \frac{n^4 - n + 1}{2n^7 - n}$ b) $\sum_{n=1}^{\infty}(-1)^n \tan\left(\frac{1}{\sqrt{n}}\right)$.

Problem 3. Determine the interval of convergence of the following power series:

a) $\sum_{n=1}^{\infty}(\arctan n)^{-n} x^n$ (6 points), b) $\sum_{n=1}^{\infty} \frac{(x-1)^{2n-1}}{n(n+1)4^n}$ (6 points).

Problem 4. a) (9 points) Express the function $f(x) = \frac{1}{\sqrt{1-2x^2}}$ as a power series centered at 0 and state the radius of convergence. Use the power series to compute $f^{(6)}(0)$ (the answer must be given as a fraction in lowest terms).

b) Find the Taylor series centered at 0 of the function $f(x) = \frac{\arctan x - x}{x^3}$. (5 points)

Problem 5. The curve $x = t^2 + 2$, $y = \frac{1}{3}t^3 - t + 2$, $t \in [0, 1]$ is revolved about the y-axis.

a) Compute the area of the resulting surface. (6 points)

b) Compute the length of the curve. (5 points)

Problem 6. Consider the simple closed curve $x = \sin t + \cos t$, $y = \cos t$, $t \in [0, 2\pi]$.

a) Find the equation of the line tangent to this curve at the point corresponding to $t = \pi/2$. (5 points)

b) Compute the area enclosed by this curve. Hint: $\int_0^{2\pi} \cos^2 x dx = \pi$. (6 points)

Problem 7. Let $f(x) = \sqrt[3]{x}$.

a) (5 points) Find the second Taylor polynomial of f centered at 8 (i.e. $T_2(\sqrt[3]{x}, 8)(x)$).

b) (5 points) Use Taylor's inequality to show that

$$|\sqrt[3]{9} - T_2(\sqrt[3]{x}, 8)(9)| \leq \frac{5}{3^4 \cdot 2^8}.$$