YOU MUST SHOW ALL OF YOUR WORK TO GET CREDIT.

1. (15 points) Let $M = \begin{bmatrix} 1 & -2 & -1 & 5 & -7 \\ 1 & -1 & 0 & 4 & -5 \\ 1 & -3 & -2 & 6 & -9 \\ 1 & 0 & 1 & 3 & -3 \end{bmatrix}$. The reduced row echelon form

for M is $A = \begin{bmatrix} 1 & 0 & 1 & 3 & -3 \\ 0 & 1 & 1 & -1 & 2 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$.

a) What is a basis for the column space of M?

b) What is a basis for the row space of M?

c) What is a basis for the null space of M?

2. (10 points) A linear transformation $L : \mathbf{R}^n \to \mathbf{R}^m$ is represented by a matrix having p pivot columns.

a) What is the dimension of the image of L?

b) What is the dimension of the kernel of L?

3. (10 points) Let

$$\mathbf{u} = \sin^2 x,$$
$$\mathbf{v} = \cos^2 x,$$
$$\mathbf{w} = (\sin x)(\cos x),$$

and you are told that these are linearly independent as functions. Let W be the vector space with basis $B = (\mathbf{u}, \mathbf{v}, \mathbf{w})$. The derivative D is a linear transformation from W to W. What is the matrix of this transformation using B for domain and range?

4. (15 points) Calculate the determinants of the following matrices. Hint on (c): If you try to do it entirely by cofactor expansion, you will never finish.

a) $\begin{bmatrix} 1 & 2 \\ 4 & 6 \end{bmatrix}$

b) $\begin{bmatrix} 1 & 2 & -1 \\ 2 & 1 & 1 \\ 1 & 2 & 2 \end{bmatrix}$

c) $\begin{bmatrix} 3 & 1 & 2 & 3 & 4 \\ 3 & 2 & 2 & 3 & 4 \\ 3 & 1 & 2 & 4 & 4 \\ 3 & 1 & 3 & 3 & 4 \\ 3 & 1 & 2 & 3 & 5 \end{bmatrix}$

BE VERY CAREFUL WITH MINUS SIGNS.   (More problems: $\to$)

5. (10 points) Find the eigenvalues of $\begin{bmatrix} 2 & 4 \\ 5 & 3 \end{bmatrix}$. Do not give more than this question asks for.

6. (20 points) The characteristic polynomial of the matrix $\begin{bmatrix} 3 & 4 & -4 \\ -4 & -7 & 8 \\ -2 & -4 & 5 \end{bmatrix}$ is $(x-1)^2(x+1)$.

   a) Give bases for the eigenspaces.

   b) Is the matrix diagonalizable? Give a reason.

   Do not give more than this question asks for.

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7. (20 points) Let $M = \begin{bmatrix} 2 & 0 & 0 \\ 6 & -4 & -6 \\ -3 & 3 & 5 \end{bmatrix}$. You are told that

$$B = \left(\begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ -2 \\ 1 \end{bmatrix} \right)$$

   is a basis for $\mathbf{R}^3$ of eigenvectors of M.

   a) What are the eigenvalues corresponding to each vector in B?

   b) Use this information to calculate M^4. (Do NOT multiply M times itself three times.)

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