Problem 1. Find the change of basis matrix from the basis $X = \{(1,1,1),(1,1,0),(1,0,0)\}$ to the basis $Y = \{(1,2,0),(0,1,-1),(1,0,1)\}$ of $\mathbb{R}^3$.

Problem 2. Consider the following function $T : P_3 \longrightarrow \mathbb{R}^2$, $T(f) = (f(0), f(1))$ (here P_3 is the space of polynomials of degree ≤ 3). Prove that T is a linear transformation. Find the kernel and range of T.

Problem 3. Let $T : \mathbb{R}^2 \longrightarrow \mathbb{R}^3$ be defined by $T(x,y) = (2x - 3y, y, x + y)$ and let $S : \mathbb{R}^3 \longrightarrow \mathbb{R}^4$ be defined by $T(x,y,z) = (x - y - z, 2y + z, x - z, 3x + y - 2z)$. Let $X = \{(1,1),(-1,1)\}$, $Y = \{(1,0,0),(1,1,0),(1,1,1)\}$, $Z = \{e_1, e_2, e_3, e_4\}$ be bases of $\mathbb{R}^2$, $\mathbb{R}^3$, $\mathbb{R}^4$ respectively. Find ${}_YT_X$, ${}_ZS_Y$ and ${}_Z(ST)_X$.

Problem 4. Let $T : \mathbb{R}^2 \longrightarrow \mathbb{R}^4$ be a linear transformation such that $T(-1,3) = (2,-1,1,0)$ and $T(2,-1) = (4,0,1,-3)$. Find the matrix of T in the standard bases of $\mathbb{R}^2$ and $\mathbb{R}^4$.

Problem 5. Write down a definition of the matrix of $T : V \longrightarrow W$ in ordered bases $X = \{v_1, ..., v_n\}$ of V and $Y = \{w_1, ..., w_m\}$ of W.