

MATH 304 - Linear Algebra

YOU MUST SHOW ALL WORK TO GET CREDIT.

1. (10 points) Consider the system of linear equations

$$\begin{aligned} 2x_1 - 3x_2 + 4x_3 - 3x_4 &= -1, \\ -3x_1 + 3x_2 - 7x_3 + x_4 &= 3, \\ x_1 + 5x_2 - 2x_3 - 4x_4 &= 11. \end{aligned}$$

- a) What is the augmented matrix of the system above?
 b) What is the coefficient matrix of the system above?
2. (20 points) Reduce

$$\begin{bmatrix} -2 & 2 & -1 & 0 & -5 \\ 4 & -4 & 1 & 0 & 11 \\ -4 & 4 & -7 & 0 & -5 \\ -2 & 2 & 2 & 1 & -4 \end{bmatrix}$$

to reduced row echelon form. BE VERY CAREFUL.

3. (15 points) The augmented matrix of a system of linear equations is

$$A = \left[\begin{array}{cccccc|c} 6 & 12 & -18 & -7 & 2 & 23 & -18 \\ -7 & -14 & 21 & 10 & -3 & -27 & 24 \\ 2 & 4 & -6 & -3 & 1 & 8 & -7 \end{array} \right]$$

and the reduced row echelon form of A is

$$R = \left[\begin{array}{cccccc|c} 1 & 2 & 3 & 0 & 0 & 4 & -1 \\ 0 & 0 & 0 & 1 & 0 & 1 & 2 \\ 0 & 0 & 0 & 0 & 1 & 3 & 1 \end{array} \right]$$

- a) What is the solution set for the system of linear equations?
 b) What are the pivot columns of A ? WARNING: This is asking for the content of the columns and not their locations.
4. (20 points) Without actually finding the solutions to the following systems, say how many solutions there are. The similarities are not a coincidence and are supposed to make your life easier. Say at least a few words to justify your answers.

a)
$$\begin{aligned} 2x_1 + x_2 + 3x_3 &= 3 \\ x_1 + x_2 + 2x_3 &= 2 \\ -x_1 + 2x_2 + x_3 &= 2 \end{aligned}$$

b)
$$\begin{aligned} 2x_1 + x_2 + 3x_3 &= 7 \\ x_1 + x_2 + 2x_3 &= 5 \\ -x_1 + 2x_2 + x_3 &= 4 \end{aligned}$$

c)
$$\begin{aligned} 2x_1 + x_2 + 3x_3 &= 0 \\ x_1 + x_2 + 2x_3 &= 0 \\ -x_1 + 2x_2 + x_3 &= 0 \end{aligned}$$

5. (10 points) A function from \mathbf{R}^n to \mathbf{R}^m is given by a coefficient matrix.
- If the matrix is a (4×3) matrix and its rank is 3, is the function one-to-one? Is it onto?
 - If the matrix is a (3×4) matrix and its rank is 3, is the function one-to-one? Is it onto?
6. (15 points) Let $A = \begin{bmatrix} 1 & 0 & -1 \\ 2 & 1 & 2 \end{bmatrix}$, let $B = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}$, and let $C = \begin{bmatrix} 1 & 2 \\ 1 & 1 \\ 1 & 3 \end{bmatrix}$.
- What is $A + B$?
 - What is $5B$?
 - What is AC ?
 - What is CA ?
7. (10 points)
- If F is a function from a vector space V to a vector space W , what does F have to satisfy in order to be a linear transformation?
 - Assume that H is a linear transformation from a vector space V to a vector space W and J is a linear transformation from the vector space W to a vector space U . Prove that the composition JH is a linear transformation.