# Quizzes for Math 304

**QUIZ 1.** A system of linear equations has augmented matrix

$$A = \begin{pmatrix} 2 & 4 & 1 & 1 & 4 \\ -1 & -2 & 0 & -1 & -1 \\ 2 & 4 & 3 & -1 & 5 \\ 1 & 2 & -1 & 1 & -1 \end{pmatrix}$$

a) Write down this system of equations;

b) Find the reduced row-echelon form of $A$;

c) What is the rank of A?

d) Solve the system of equations found in a).

**QUIZ 2.** a) Is the matrix

$$\begin{pmatrix} 1 & 2 & 3 & 4 \\ -1 & 1 & 0 & 2 \\ 2 & 0 & 1 & 1 \end{pmatrix}$$

one-to-one? Explain your answer.

b) Is the matrix

$$\begin{pmatrix} 1 & -1 & 1 & -1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & -1 & -1 \\ 1 & -1 & -1 & 1 \end{pmatrix}$$

onto? Is it one-to-one? Explain your answer.

c) State a definition of a linear transformation.

**QUIZ 3.** a) Define linear combination of vectors $v_1, ..., v_n$.

b) The function $T : \mathbb{R}^3 \longrightarrow \mathbb{R}^2$, $T(a,b,c) = (a-b+c, 2a+c)$ is a linear transformation. What is the matrix of $T$?

c) Compute the product

$$C = \begin{pmatrix} 1 & 1 \\ 0 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} 1 & 2 & 3 \\ -2 & 1 & 0 \end{pmatrix}$$

Is $C$ 1-1? Answer this question without any computations.

**QUIZ 4.** Let $A = \begin{pmatrix} 3 & 2 & 0 \\ 2 & 1 & 1 \\ 5 & 3 & 2 \end{pmatrix}$.

a) Find $A^{-1}$.

b) Express $A$ as a product of elementary matrices.

c) Suppose that $B$ is a matrix such that $AB = \begin{pmatrix} 1 & 2 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix}$. Is $B$ invertible? Explain your answer.

**QUIZ 5.** a) The vectors $v_1, ..., v_n$ of a vector space $V$ are linearly independent iff ... (state all three equivalent conditions).

b) Are the vectors $v_1 = (1,0,2,1)$, $v_2 = (2,1,1,1)$, $v_3 = (-1,-2,4,1)$ linearly independent? If no, find a dependence relation among them.

c) Is $(1,-1,1,-1)$ in the span of $\{v_1, v_2, v_3\}$?

**QUIZ 6.** a) Define a basis of a vector space $V$.

b) Find a basis of $\text{span}\{(1,0,1),(1,3,-2),(1,1,0)\}$.

c) Let $T : V \longrightarrow W$ be a linear transformation and $v_1, ..., v_n \in V$. Prove that if $T(v_1), ..., T(v_n)$ are linearly independent then $v_1, ..., v_n$ are also linearly independent.

**QUIZ 7.** a) Define the change of basis matrix from the basis $X = \{v_1, ..., v_n\}$ to the basis $Y = \{w_1, ..., w_n\}$ of a vector space $V$.

b) Find the transition matrix from from the basis $X = \{e_1, e_2, e_3\}$ to the basis $Y = \{(1,1,1), (1,2,1), (-1,-3,2)\}$ of $\mathbb{R}^3$.

c) Find the coordinates of the vector $(1,2,3)$ in the basis $Y$.

**QUIZ 8.** a) Find the change of basis matrix from the basis $\{(1,1,0), (1,0,1), (0,1,1)\}$ to the basis $\{(1,0,0), (1,1,0), (1,1,1)\}$ of $\mathbb{R}^3$.

b) State a definition of the determinant.

c) Compute the determinanat of the matrix

$$\begin{pmatrix} 0 & 1 & 3 \\ -2 & -3 & -5 \\ 4 & -4 & 4 \end{pmatrix}.$$

**QUIZ 9.** a) State a definition of an eigenvector and an eigenvalue of a matrix $A$ (or a linear transformation $T : V \longrightarrow V$, if you prefer).

b) Let $A = \begin{pmatrix} 3 & -2 \\ 2 & -1 \end{pmatrix}$

- Find the eigenvalues of $A$.
- Find the eigenvectors of $A$.

**QUIZ 10.** a) State a definition of an inner product on a vector space $V$.

b) The inner product $< \, , \, >$ on $\mathbb{R}^2$ has matrix $Q = \begin{pmatrix} 3 & 3 \\ 3 & 5 \end{pmatrix}$ in the standard basis of $\mathbb{R}^2$.

- Compute $< (1,2), (2,1) >$.
- Find a vector orthogonal to $(1,0)$.