Exam I, Math 323

February 18, 2004

Problem 1. a) Compute the length of the vector $\mathbf{v} = 2\mathbf{i} - 3\mathbf{j} + \mathbf{k}$. (**2 points**)

b) Find the angles of the triangle with vertices $A(0,0)$, $B(0, 3+\sqrt{3})$, $C(3, \sqrt{3})$. (**4 points**)

c) Find a unit vector orthogonal to $< 3, 4 >$. (**2 points**)

d) Let $\mathbf{a} =< 1, 0, 1 >$ and $\mathbf{b} =< 1, 1, 0 >$. Find vectors $\mathbf{u}$, $\mathbf{w}$ such that $\mathbf{u}$ is parallel to $\mathbf{a}$, $\mathbf{w}$ is orthogonal to $\mathbf{a}$ and $\mathbf{b} = \mathbf{u} + \mathbf{w}$. (**4 points**)

Problem 2. a) Compute $(2\mathbf{i} + \mathbf{j} - \mathbf{k}) \times (\mathbf{i} - 2\mathbf{j} + 3\mathbf{k})$. (**3 points**)

b) Compute the determinant $\begin{vmatrix} 1 & 1 & 1 \\ 1 & 2 & 1 \\ 1 & 1 & 2 \end{vmatrix}$. (**2 points**)

c) Find the volume of the parallelepiped determined by the vectors $< 0, 0, 1 >$, $< 1, 0, 1 >$, $< 1, 1, 1 >$. (**3 points**)

d) What is the area of the triangle with vertices $(0,0,0), (1,0,1), (1,1,1)$? (**3 points**)

Problem 3. a) Find the radius and the center of the sphere $x^2 + y^2 + z^2 = x - y + z$. (**3 points**)

b) Find a parametric and symmetric equations of the line of intersection of the planes $2x - y - z = 0$ and $x - 2y + z = 0$. (**5 points**)

c) Find an equation of the plane containing points $(1,0,1), (0,1,1), (1,1,0)$. (**4 points**)

Problem 4. a) Define the curvature of the curve $\mathbf{r}(t)$. (**3 points**)

b) Compute the velocity, speed, acceleration, unit tangent vector, unit normal vector and curvature of the curve $\mathbf{r}(t) =< 2t - \sin 2t, -\cos 2t, 4 \sin t >$. (**6 points**)
Hint: $\cos 2x = 2\cos^2 x - 1$

c) Find an arc-length (natural) parametrization of the curve (**4 points**)

$$\mathbf{r}(t) =< t\sin t, t\cos t, \frac{2\sqrt{2}}{3}t^{3/2} > .$$

d) A particle moves in the space with acceleration $\mathbf{a}(t) =< 2, 6t, 12t^2 >$. At the time $t = 1$ the particle is at the point $(3, 2, 2)$ and has velocity $< 3, 4, 5 >$. Find the position of the particle at the time $t = 0$. (**3 points**)

Problem 5. a) Find cylindrical and spherical coordinates of the point $(1, 1, \sqrt{6})$. (**3 points**)

b) Find spherical coordinates of the point whose cylindrical coordinates are $(1, \pi/6, 1)$. (**3 points**)

c) A plane curve in polar coordinates has equation $r = \cos\theta$. Find the curvature of this curve as a function of θ. (**4 points**)

Problem 6. a) Find the domain of the function $f(x, y) = \ln(x^2 + x + y^2 - 1)$. (**2 points**)

b) Describe the level curves of the function $f(x, y) = e^{xy}$. (**3 points**)

c) Explain why the limit $\lim_{(x,y)\to(0,0)} \frac{x^2 + y^3}{x^2 + y^2}$ does not exist. (**4 points**)

d) Let $f(x, y, z) = \frac{x^2 y}{x^2 + y^2 + z^2}$ for $(x, y, z) \neq (0, 0, 0)$ and $f(0, 0, 0) = a$. Find all values a such that this function is continuous. (**4 points**)

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The following problem is optional. You may earn extra points, but work on this problem only after you are done with the other problems

Problem 7. a) The acceleration and velocity of a parametric curve $\mathbf{r}(t)$ are always orthogonal. Prove that the speed of this curve is constant.(**8 points**)

b) A plane curve $\mathbf{r}(t)$ has constant curvature $k > 0$. Prove that this curve is a circle. (**10 points**)

Hint for b). Work with arc-length parametrization. Show that $k^2\mathbf{r}(s) + \mathbf{a}(s)$ is constant.