Exam II, Math 323

March 24, 2004

Problem 1. a) Let $f(x,y) = (\sqrt[3]{x} + \sqrt[3]{y})^3$ and $\mathbf{u} =< 1/\sqrt{2}, 1/\sqrt{2} >$. Compute, using the definition, the directional derivative $D_{\mathbf{u}}f(0,0)$ of the function f at the point $(0,0)$ in the direction of the unit vector $\mathbf{u}$. (**8 points**)

b) (optional) As in a), one can compute the partial derivatives $\frac{\partial f}{\partial x}(0,0) = 1 = \frac{\partial f}{\partial y}(0,0)$ (this, in fact, was a problem on a quiz). Compare $D_{\mathbf{u}}f(0,0)$ and $\nabla f(0,0)\cdot\mathbf{u}$. Can you explain why this does not contradict one of our theorems. (**4 points**)

Problem 2. Find the equation of the plane tangent to the surface

$$x^2 + y^2 + z^2 - xyz = 2$$

at the point $(1,1,0)$. (**8 points**)

Problem 3. a) State the Implicit Function Theorem. (**5 points**)

b) The Implicit Function Theorem implies that the surface

$$x^2 + y^2 + z^2 - xyz = 2$$

is a graph of a function $z = g(x,y)$ near the point $(1,1,0)$ (why?). Compute the gradient $\nabla g(1,1)$. (**9 points**)

c) Let $h(s,t) = F(x(s,t), y(s,t))$. Compute $\frac{\partial h}{\partial t}(0,1)$ knowing that

$$x(0,1) = 1,\ y(0,1) = 2,\ \frac{\partial x}{\partial t}(0,1) = -1,\ \frac{\partial y}{\partial t}(0,1) = 1,\ \frac{\partial F}{\partial x}(1,2) = 3,\ \frac{\partial F}{\partial y}(1,2) = 2$$

(**9 points**)

Problem 4. Find largest and smallest values of the function $f(x,y) = x + y$ subject to the condition $x^4 + 4xy + 2y^2 + 1 = 0$. (**12 points**)

Problem 5. Let $f(x, y) = x^2 - y^2 - x^2y^2$. Find the largest and smallest values of f on the region D contained inside the circle $x^2 + y^2 = 1$ and above the x-axis. (**12 points**)

Problem 6. Classify all critical points of the function $f(x, y) = x^2 + 2xy^4 - 4xy^2$. (**12 points**) (Hint: there are 5 such points).

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The following problems are optional. You may earn extra points, but work on these problems only after you are done with the other problems

Problem 7. Find a non-zero vector tangent at the point $(1, 1, 1)$ to the curve of intersection of the surfaces $x^4 + y^4 + z^4 = 3$ and $x + y - 2z = 0$. (**10 points**)

Problem 8. Suppose that a continuously differentiable function $f(x, y)$ satisfies the equation

$$x\frac{\partial f}{\partial y}(x, y) - 3y\frac{\partial f}{\partial x}(x, y) = 0$$

Show that, for every constant c, the function f is constant on the curve $x^2+3y^2 = c$. (**15 points**)