Exam III, Math 323

April 28, 2004

Problem 1. a) Compute $\int\int_R \frac{1}{3+x-x^2}dxdy$, where R is the region bounded by the parabola $y = x^2 - 2$, the line $y = x + 1$ and the vertical lines $x = -1$ and $x = 2$. (**7 points**)

b) Compute $\int\int\int_T 4xyzdxdydz$, where T is the solid bounded by the parabolic cylinder $y = 1 - x^2$ and the planes $y = 0$, $z = 1$, $z = 0$. (**8 points**)

c) Compute $\int_0^1 \int_{\sqrt{y}}^1 3e^{x^3}dxdy$. Hint: reverse the order of integration. (**8 points**)

Problem 2. a) Compute $\int\int_R 2\cos(x^2+y^2)dxdy$, where R is the region in the first quadrant bounded by the curve $r = \sqrt{\pi + \theta}$ (in polar coordinates). (**7 points**)

b) Compute $\int\int\int_T 4z\sqrt{1+(x^2+y^2+z^2)^2}dxdydz$, where T is the part of the ball $x^2+y^2+z^2 \leq 1$ in the first octant. Hint: The Jacobian of the spherical coordinates is $\rho^2 \sin\phi$. (**8 points**)

Problem 3. a) Compute $\int_C e^{4x^2-y^2}ds$, where C is the piece of the line $y = 2x$ which lies between $x = 0$ and $x = 1$. (**7 points**)

b) Compute $\int_C y^2dx + \cos x dy$, where C is the part of the graph of $y = \sin x$ between $x = 0$ and $x = \pi$ oriented from the point $(0, 0)$ to the point $(\pi, 0)$. (**7 points**)

c) Compute $\int_C \mathbf{F} \cdot \mathbf{T}ds$, where C is the helix $\mathbf{r}(t) =< \cos t, \sin t, t >$, $t \in [0, \pi]$ and $\mathbf{F}$ is the vector field $\mathbf{F}(x, y, z) =< -y, x, z - x^2 - y^2 >$. (**8 points**)

Problem 4. a) Is the vector field $\mathbf{F}(x, y) =< xy, y - x^2 >$ conservative? Explain. (**7 points**)

b) Find a potential for the vector field $\mathbf{F}(x, y) =< \cos x \sin y, \sin x \cos y + 2y >$. (**8 points**)

****

****

The following problems are optional. You may earn extra points, but work on these problems only after you are done with the other problems

Problem 5. Find the area of the surface $x = \cos u$, $y = \sin u$, $z = u + v$, where $0 \le u \le \pi$ and $0 \le v \le 1$. (**8 points**)

Problem 6. Use Green's theorem to compute

$$\int_C (\tan(x^2) - y^3)dx + (e^{e^y} + 3x^2)dy,$$

where C is the unit circle oriented counterclockwise. (**8 points**)

Problem 7. The change of variables $x = x(r,t,\theta) = \frac{r}{t}\cos\theta$, $y = y(r,t,\theta) = \frac{r}{t}\sin\theta$, $z = z(r,t,\theta) = r^2$ transforms the box $B = \{(r,t,\theta) : 1 \le r \le 2, 1 \le t \le 2, 0 \le \theta \le 2\pi\}$ onto the solid R between the paraboloids $z = x^2 + y^2$, $z = 4(x^2 + y^2)$ and the planes $z = 1$, $z = 4$. Express the volume of R as a triple integral in the variables r, t, θ and evaluate it. (**8 points**)