

Quizzes for Math 323

QUIZ 1. a) Find $|\mathbf{a}|$, $|-2\mathbf{b}|$, $|\mathbf{a} - \mathbf{b}|$, $3\mathbf{a} - 2\mathbf{b}$, where $\mathbf{a} = \langle 3, 4 \rangle$ and $\mathbf{b} = \langle -4, 3 \rangle$.

b) Find a vector of length 5 with the same direction as $7\mathbf{i} - 3\mathbf{j}$.

QUIZ 2. a) Let $\mathbf{v} = \langle 1, \sqrt{2}, 1 \rangle$ and $\mathbf{w} = \langle 1, 0, 1 \rangle$. Compute $\mathbf{v} \cdot \mathbf{w}$, $\mathbf{v} \times \mathbf{w}$ and the angle between \mathbf{v} and \mathbf{w} . What is the area of the parallelogram determined by \mathbf{v} and \mathbf{w} ?

b) Compute $\begin{vmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 1 & 0 \end{vmatrix}$. What is the volume of the parallelepiped determined by vectors $\langle 1, 0, 1 \rangle$, $\langle 0, 1, 1 \rangle$ and $\langle 1, 1, 0 \rangle$?

QUIZ 3. a) Find equation of the plane passing through point $P(3, -5, 1)$ and orthogonal to $\mathbf{v} = \mathbf{i} + \mathbf{j} - \mathbf{k}$.

b) Find parametric and symmetric equations of the line through $(1, -1, 2)$ and $(3, 2, -1)$.

QUIZ 4. a) Find the velocity, speed, acceleration and the unit tangent vector of the curve $\mathbf{r}(t) = \langle \sin t, \cos t, t^2/2 \rangle$. Find the curvature of this curve at the point $\mathbf{r}(0)$.

b) What does it mean that the parametrization $\mathbf{r}(t)$ is arc-length (natural)?

QUIZ 5. a) Use spherical coordinates to compute $\lim_{(x,y,z) \rightarrow (0,0,0)} \frac{x^3 + y^3}{x^2 + y^2 + z^2}$.

b) Explain why $\lim_{(x,y,z) \rightarrow (0,0,0)} \frac{x^2 + xy + z^2}{x^2 + y^2 + z^2}$ does not exist.

QUIZ 6. a) Using the definition, compute the partial derivatives at $(0, 0)$ of the

function

$$f(x, y) = (\sqrt[3]{x} + \sqrt[3]{y})^3.$$

b) Find critical points of $f(x, y) = x^2 + xy - y^2 + 1$.

QUIZ 7. a) Find the distance from $(0, 0, 0)$ to the plane $2x + 3y - z = 3$;

b) Let $w = x^2 + yz + xy$, $x = \sin t$, $y = \cos t$, $z = e^t$. Compute $\frac{dw}{dt}(0)$.

QUIZ 8. a) Find equation of the plane tangent to the surface

$$x^3 + y^3 + x^3 - xy - yz - xz = 0$$

at the point $(1, 1, 1)$.

b) Find minimum and maximum of the function $f(x, y) = x + y$ subject to the condition $x^2 + xy + y^2 = 3$.

QUIZ 9. Compute the integral $\int \int_R xy dA$, where R is the triangle with vertices $(0, 0)$, $(1, 0)$, $(1, 2)$.

QUIZ 10. Find the area of the part of the surface $z = 9 - x^2 - y^2$ above the plane $z = 5$.

QUIZ 11. a) Compute $\int_C y ds$, where C is the semicircle $x^2 + y^2 = 1$, $y \geq 0$.

b) Find a potential for the vector field $F(x, y) = \langle e^y, xe^y \rangle$.

QUIZ 12. a) State Green's Theorem.

b) Compute

$$\oint_C (x + y^2) dx + (x^2 + y) dy,$$

where C is the boundary of the square with vertices $(\pm 1, \pm 1)$.