Exam I, Math 323

February 26, 2008

Problem 1. a) Compute the length of the vector $\mathbf{v} = 2\mathbf{i} - 3\mathbf{j} + \mathbf{k}$. (**4 points**)

b) Find the angles $\angle A$, $\angle B$ of the triangle with vertices $A(0,0)$, $B(0, 3+\sqrt{3})$, $C(3, \sqrt{3})$. What is $\angle C$? (**6 points**)

c) Find a unit vector orthogonal to $< 3, 4 >$. (**4 points**)

d) Let $\mathbf{a} =< 1, 0, 1 >$ and $\mathbf{b} =< 1, 1, 0 >$. Find vectors $\mathbf{u}$, $\mathbf{w}$ such that $\mathbf{u}$ is parallel to $\mathbf{a}$, $\mathbf{w}$ is orthogonal to $\mathbf{a}$ and $\mathbf{b} = \mathbf{u} + \mathbf{w}$. (**6 points**)

Problem 2. a) Compute $(2\mathbf{i} + \mathbf{j} - \mathbf{k}) \times (\mathbf{i} - 2\mathbf{j} + 3\mathbf{k})$. (**5 points**)

b) Compute the determinant $\begin{vmatrix} 1 & 1 & 1 \\ 1 & 2 & 1 \\ 1 & 1 & 2 \end{vmatrix}$. (**5 points**)

c) Find the volume of the parallelepiped determined by the vectors $< 0, 0, 1 >$, $< 1, 0, 1 >$, $< 1, 1, 1 >$. (**5 points**)

d) What is the area of the triangle with vertices $(0,0,0), (1,0,1), (1,1,1)$? (**5 points**)

Problem 3. a) Find the radius and the center of the sphere $x^2 + y^2 + z^2 = x - y + z$. (**5 points**)

b) Find parametric and symmetric equations of the line of intersection of the planes $2x - y - z = 0$ and $x - 2y + z = 0$. (**5 points**)

c) Find an equation of the plane containing points $(1,0,1), (0,1,1), (1,1,0)$. (**5 points**)

Problem 4. a) What is the curvature of a circle of radius R?. Just write down the answer, no work need to be shown.(**3 points**)

b) Compute the velocity, speed, acceleration, unit tangent vector, unit normal vector and curvature of the curve $\mathbf{r}(t) =< 2t - \sin 2t, -\cos 2t, 4\sin t >$ at the point corresponding to $t = \pi/2$. (**7 points**)

c) Find an arc-length (natural) parametrization of the curve (**6 points**)

$$\mathbf{r}(t) =< t\sin t, t\cos t, \frac{2\sqrt{2}}{3}t^{3/2} > .$$

In other words, if s is the arc-length function in terms of t, find $\mathbf{r}(s)$.

d) A particle moves in the space with acceleration $\mathbf{a}(t) =< 2, 6t, 12t^2 >$. At the time $t = 1$ the particle is at the point $(3, 2, 2)$ and has velocity $< 3, 4, 5 >$. Find the position of the particle at the time $t = 0$. (**6 points**)

Problem 5. a) Find cylindrical and spherical coordinates of the point $(1, 1, \sqrt{6})$. (**5 points**)

b) Find spherical coordinates of the point whose cylindrical coordinates are $(1, \pi/6, 1)$. (**5 points**)

c) A plane curve in polar coordinates has equation $r = \cos\theta$. First find the parametric equation in rectangular coordinates with parameter θ, then find the curvature of this curve as a function of θ. (**6 points**)

Problem 6. Consider the surface with equation $y^2 - 4x^2 - 25z^2 = 100$.

(a) (6 points) Its intersection with the

(a) XY plane is

an ellipse	a hyperbola	empty	something else

(b) XZ plane is

an ellipse	a hyperbola	empty	something else

(c) YZ plane is

an ellipse	a hyperbola	empty	something else

(b) (2 points) This surface is

ellipsoid	hyperboloid of 1 sheet	hyperboloid of 2 sheets	paraboloid	something else

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The following problem is optional. You may earn extra points, but work on this problem only after you are done with the other problems

Problem 7. The acceleration and velocity of a parametric curve $\mathbf{r}(t)$ are always orthogonal. Prove that the speed of this curve is constant.(**8 points**)