# Exam III, Math 323

April 29, 2008

**Problem 1.** The function $f(x,y) = x^2 + 2xy^4 - 4xy^2$ has five critical points $(1,1)$, $(1,-1)$, $(0,\sqrt{2})$, $(0,-sqrt2)$ and $(0,0)$. Determine the type of each of these critical points. Provide all the necessary calculatuions. (**15 points**)

**Problem 2.** a) Compute $\int\int_R \frac{1}{3+x-x^2}dxdy$, where $R$ is the region bounded by the parabola $y = x^2 - 2$, the line $y = x + 1$ and the vertical lines $x = -1$ and $x = 2$. (**10 points**)

b) Compute $\int\int\int_T 4xyzdxdydz$, where $T$ is the solid bounded by the parabolic cylinder $y = 1 - x^2$ and the planes $y = 0$, $z = 1$, $z = 0$. (**10 points**)

c) Compute $\int_0^1 \int_{\sqrt{y}}^1 3e^{x^3}dxdy$. Hint: reverse the order of integration. (**10 points**)

**Problem 3.** a) Compute $\int\int_R 2\cos(x^2+y^2)dxdy$, where $R$ is the region in the first quadrant bounded by the curve $r = \sqrt{\pi+\theta}$ (in polar coordinates). (**10 points**)

b) Compute $\int\int\int_T 4z\sqrt{1+(x^2+y^2+z^2)^2}dxdydz$, where $T$ is the part of the ball $x^2+y^2+z^2 \leq 1$ in the first octant. Hint: The Jacobian of the spherical coordinates is $\rho^2\sin\phi$. (**10 points**)

**Problem 4.** a) Compute $\int_C e^{4x^2-y^2}ds$, where $C$ is the piece of the line $y = 2x$ which lies between $x = 0$ and $x = 1$. (**10 points**)

c) Compute $\int_C \mathbf{F}\cdot\mathbf{T}ds$, where $C$ is the helix $\mathbf{r}(t) =< \cos t, \sin t, t >$, $t \in [0,\pi]$ and $\mathbf{F}$ is the vector field $\mathbf{F}(x,y,z) =< -y, x, z - x^2 - y^2 >$. (**10 points**)

**Problem 5.** Find the area of the surface $x = \cos u$, $y = \sin u$, $z = u + v$, where $0 \leq u \leq \pi$ and $0 \leq v \leq 1$. (**15 points**)

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The following problem is optional. You may earn extra points, but work on this problem only after you are done with the other problems

**Problem 6.** The change of variables $x = x(r, t, \theta) = \frac{r}{t} \cos\theta$, $y = y(r, t, \theta) = \frac{r}{t} \sin\theta$, $z = z(r, t, \theta) = r^2$ transforms the box $B = \{(r, t, \theta) : 1 \leq r \leq 2, 1 \leq t \leq 2, 0 \leq \theta \leq 2\pi\}$ onto the solid $R$ between the paraboloids $z = x^2 + y^2$, $z = 4(x^2 + y^2)$ and the planes $z = 1$, $z = 4$. Express the volume of $R$ as a triple integral in the variables $r, t, \theta$ and evaluate it. (**8 points**)