Quizzes for Math 323

QUIZ 1. a) Find $|\mathbf{a}|, |-2b|, |\mathbf{a} - \mathbf{b}|, 3\mathbf{a} - 2\mathbf{b}$, where $\mathbf{a} = <3, 4 > \text{and } \mathbf{b} = -4i + 3j$.

b) Let $\mathbf{v} = \langle 1, \sqrt{2}, 1 \rangle$ and $w = \langle 1, 0, 1 \rangle$. Compute $\mathbf{v} \cdot \mathbf{w}$, $\mathbf{v} \times \mathbf{w}$ and the angle between \mathbf{v} and \mathbf{w} . What is the area of the parallelogram determined by \mathbf{v} and \mathbf{w} ?

QUIZ 2. a) Find equation of the plane passing through point P(3, -5, 1) and orthogonal to $\mathbf{v} = \mathbf{i} + \mathbf{j} - \mathbf{k}$.

b) Find parametric and symmetric equations of the line through (1, -1, 2) and (3, 2, -1).

c) Compute $\begin{vmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 1 & 0 \end{vmatrix}$. What is the volume of the parallelepiped determined by vectors < 1, 0, 1 >, < 0, 1, 1 > and < 1, 1, 0 >?

QUIZ 3. a) Find the velocity, speed, acceleration, the unit tangent vectorand curvature of the curve $\mathbf{r}(t) = <\sin t, \cos t, t^2/2 >$ as functions of t. Find the curvature and unit tangent and normal vectors of this curve at the point $\mathbf{r}(0)$.

b) The cylindrical coordinates of a point are $(1, \pi/4, -\sqrt{3})$. What are the spherical coordinates of this point?

QUIZ 4. a) Explain why $\lim_{(x,y,z)\to(0,0,0)} \frac{x^2 + xy + z^2}{x^2 + y^2 + z^2}$ does not exist.

b) Find critical points of $f(x, y) = x^2 + xy - y^2 + 1$.

c) Find the distance from (0, 0, 0) to the plane 2x + 3y - z = 14.

QUIZ 5. a) Let f(x, y, z), x(s, t), y(s, t), z(s, t) be continuously differentiable

functions. Compute $\frac{\partial f}{\partial t}(1,2)$ knowing that

$$x(1,2) = 3, \ y(1,2) = 1, \ z(1,2) = 2, \frac{\partial x}{\partial t}(1,2) = -1, \ \frac{\partial y}{\partial t}(1,2) = 0, \ \frac{\partial z}{\partial t}(1,2) = 1,$$
$$\frac{\partial f}{\partial x}(3,1,2) = 5, \ \frac{\partial f}{\partial y}(3,1,2) = 3, \ \frac{\partial f}{\partial z}(3,1,2) = 1.$$

b) Find equation of the plane tangent to the surface

$$x^3 + y^3 + x^3 - xy - yz - xz = 0$$

at the point (1, 1, 1).

c) Find the largest value of $f(x, y) = x + \sqrt{3}y$ on the closed disk $x^2 + y^2 \le 1$.

QUIZ 6. Classify critical points of the function $f(x, y) = x^3 - 3x^2y - 6y^2 + 24y$.

QUIZ 7. a) Compute the integral $\int \int_R xy dx dy$, where *R* is the triangle with vertices $(0,0), (1,0), (1,\sqrt{3})$.

b) Observe that the region R from a) is radially simple and set up the integral in a) as iterated (double) integral in polar coordinates.

QUIZ 8. a) Compute $\int \int \int_T x^2 dx dy dz$, where T is the tetrahedron bounded by the coordinate planes and the plane x + y + z = 1.

b) Set up a triple integral in spherical coordinates which is equal to $\int \int \int_V z dx dy dz$, where V is the solid bounded by the hemisphere $x^2 + y^2 + z^2 = a^2$, z > 0 and the xy-plane.

QUIZ 9. a) Find the area of the part of the surface $z = 9 - x^2 - y^2$ above the plane z = 5.

b) Let
$$\Psi(u, w) = (\sqrt{\frac{u}{w}}, \sqrt{uw})$$
. Compute the Jacobian of Ψ .