Homework

due on Monday, February 28

Read carefully the notes on sets linked on the course web page and sections 5.1-5.2 in the book. Solve the following problems.

Problem 1. Let A, B, C be sets.

a) Build a membership table to prove the first De Morgan's law: $A \setminus (B \cup C) = (A \setminus B) \cap (A \setminus C)$.

b) Prove the second De Morgan's law as follows. Express each side of the equality

$$A \setminus (B \cap C) = (A \setminus B) \cup (A \setminus C)$$

in terms of the operations $+, \cdot$, where $+$ denotes symmetric difference and $\cdot$ denotes intesection. Then use basic algebraic manipulations to prove that both sides are equal.

c) Prove that $[A \setminus (B \div C)] \cap B = A \cap B \cap C$ in two ways using the method of a) and the method of b).

Problem 2. Prove by induction on n that

$$A \setminus (A_1 \cup A_2 \cup \ldots \cup A_n) = (A \setminus A_1) \cap (A \setminus A_2) \cap \ldots \cap (A \setminus A_n).$$

Formualte and prove a similar generalization of the second De Morgan's law.

Problem 3. This problem is **optional**, you may earn extra credit. Prove that for any natural number n, an elemant x belongs to the set $A_1 \div A_2 \div \ldots \div A_n$ if and only if x belongs to an odd number of the sets $A_1, A_2, \ldots, A_n$.