Homework

due on Friday, March 11

Read carefully the notes about functions linked on the course web page and sections 5.4 and 9.1 in the book. Solve the following problems.

Problem 1. Let $f : A \longrightarrow B$ be a function.

a) Prove that if S, T are subsets of A then $f(S \cup T) = f(S) \cup f(T)$.

b) Prove that if S, T are subsets of B then $f^{-1}(S \cap T) = f^{-1}(S) \cap f^{-1}(T)$.

c) State and prove a result relating $f^{-1}(S \div T)$ and $f^{-1}(S) \div f^{-1}(T)$

d) Prove that if S, T are subsets of A then $f(S) \setminus f(T) \subseteq f(S \setminus T)$. Show by example that the equality does not always hold.

Problem 2. Let $f : A \longrightarrow B$ be a function. Prove that f is surjective if and only if there is a function $g : B \longrightarrow A$ such that $fg = \mathrm{id}_B$.