Homework

due on Monday, March 14

Read carefully section 6.1 in the book. Solve the following problems.

Problem 1. In class we started a discussion about constructing rational numbers. We considered a set $A = \mathbb{Z} \times (\mathbb{Z} \setminus \{0\})$ and we declered two elements (a, b) and (p, q) of A to be in relation R if $aq = pb$. Prove that R is an equivalence relation on A. What elements belong to the equivalence class of $(0, 1)$?

Problem 2. Let R be a relation on a set A. Recall the following terminology.

R is **reflexive** if xRx for every $x \in A$.

R is **irreflexive** (or **antireflexive**) if xRx is false for every $x \in A$.

R is **symmetric** if, for any $x, y \in A$, xRy implies yRx.

R is **antisymmetric** if, for any $x, y \in A$, xRy and yRx imply $x = y$.

R is **asymetric** if, for any $x, y \in A$, xRy implies that yRx does not hold.

R is **transitive** if, for all $x, y, z \in A$, xRy and yRz imply xRz.

Remark. Some use the name *weakly antisymmetric* for what above is called antisymmetric and the name *antisymmetric* for what above is called asymetric. But the above terminology seems to be used by majority of recent sources.

a) Let R be the relation $a|b$ on the set $\mathbb{N}$ of natural numbers (in other words, aRb means $a|b$). Which of the above properties hold for R. Carefully justify your answer for each property.

b) Let A be the set of all lines in the plane. Let R be the relation of being parallel (i.e. $l_1 R l_2$ if the line l_1 is parallel to the line l_2). Prove that R is an equivalence relation.

c) Let R_1 and R_2 be two equivalence relations on a set A (recall that this menas in particular that R_1 and R_2 are subsets of $A \times A$). Prove that $R_1 \cap R_2$ is also an equivalence relation.