Homework

due on Friday, March 18

Read carefully sections 6.1-6.4 in the book. Read the notes about relations linked on the course page. Solve the following problems.

Problem 1. Let n be a natural numbers whose decimal representation is $a_k a_{k-1} \dots a_0$. Define $A(n) = a_0 - a_1 + a_2 - \dots + (-1)^k a_k$ (i.e. $A(n)$ is the alternating sum of digits of n). Prove that $n \equiv A(n) \pmod{11}$ (mimic the proof from class of similar congruences modulo 3 and 9). Using this result find the remainder obtained when 123456789987654321 is divided by 11. Can you generalize this example?

Problem 2. a) Find all positive integers m such that $100 \equiv -5 \pmod{m}$.

b) Find the remainder when 3^{1000} is divided by 7. Hint: $3^{1000} = 9^{500} \equiv 2^{500} \pmod{7}$ and $2^3 \equiv 1 \pmod{7}$.

c) Suppose that $a \equiv b \pmod{m}$, where a, b, m are integers. Let d be a positive integer such that $d|a$, $d|b$, $d|m$. Prove that $a/d \equiv b/d \pmod{m/d}$.