Homework

due on Tuesday, April 5

Read carefully sections 6.1-6.4 in the book. Read the notes about congruences linked on the course page. Solve the following problems.

Problem 1. Let p be a prime number. Prove that there are no positive integers m, n such that $m^2 = pn^2$. Conclude that there is no rational number q such that $q^2 = p$.

Problem 2. In class we proved that if a, b, c are integers such that $\gcd(a, c) = 1$ and $c|ab$ then $c|b$. Use this to prove that if k, m, n are integers such that $\gcd(m, n) = 1$, $m|k$, $n|k$ then $mn|k$.

Problem 3. a) Prove that if m is an integer then either $m^2 \equiv 0 \pmod 4$ or $m^2 \equiv 1 \pmod 4$

b) Prove that if $n \equiv 3 \pmod 4$ then n is not a sum of two squares of integers.

Problem 4. Let a, b be natural numbers and let $d = \gcd(a, b)$.

a) Prove that a/d and b/d are relatively prime.

b) Let c be an inetegr such thta $a|c$ and $b|c$. Prove that $\frac{a}{d}\frac{b}{d}|\frac{c}{d}$. Conclude that $\frac{ab}{d}|c$.

c) Use b) to prove that ab/d is the smallest natural number which is divisible by a and b. It is called the **least common multiple** of a and b.