Homework

due on Friday, May 6

Read carefully sections 10.1-10.4 and 12.1-12.2 in the book. Solve the following problems.

Problem 1. a) Follow the proof from class to find the decimal expansion of the number 3/52.

b) Write the number $1.222357357357\ldots$ as a fraction.

Problem 2. Let a, n be positive integrs such that $\gcd(a, n) = 1$. Prove that $a^k \equiv 1 \pmod{n}$ for some positive integer k. (Hint. In class we proved a special case when $a = 10$. Modify the proof to the general case).

Problem 3. Let $a_n = n^2/2^n$.

a) Compute the limit of the sequence $b_n = a_{n+1}/a_n$.

b) Prove that $a_n > a_{n+1}$ for every $n \geq 3$.

c) Prove that a_n converges. Then use a) to prove that $\lim_{n\to\infty} a_n = 0$.

Problem 4. Use Pinching Theorem to compute the limit of the sequence $a_n = \sqrt[n]{2^n + 3^n + 5^n}$. Hint: Prove that $5^n < 2^n + 3^n + 5^n < 3 \cdot 5^n$.