Homework

due on Wednesday, May 11

Read carefully Chapter 13 and Appendix F in the book. Solve the following problems.

Problem 1. a) Let $f : A \longrightarrow B$ be surjective. Let C be a subset of B. Prove that there is a surjective function from A onto C.

b) Recall that $\mathcal{P}(X)$ is the set of all subsets of a set X. Define a function $h : \mathcal{P}(\mathbb{N}) \longrightarrow (0, \infty)$ as follows: for a subset X of $\mathbb{N}$ set $h(X) = 0.a_1a_2\ldots$, where $a_i = 1$ if $i \in X$ and $a_i = 0$ of $i \notin X$. Prove that h is injective.

c) Use a) and b) and the fact that there is no surjective function from X onto $\mathcal{P}(X)$ to prove that there is no surjective function from $\mathbb{N}$ onto $\mathbb{R}$.

Problem 2. a) Let x, y be postitive real numbers. Suppose that $\lfloor 2^k x \rfloor = \lfloor 2^k y \rfloor$ for $k = 1, 2, \ldots$. Prove that $x = y$.

b) Assign to a positive real number x the subset $g(x) = \{k + \lfloor 2^k x \rfloor : k \in \mathbb{N}\}$ of $\mathbb{N}$. Prove that g is an injective function from $(0, \infty)$ to $\mathcal{P}(\mathbb{N})$.

c) Using problems 1b), 2b) and the Cantor-Bernstein Theorem, what can you say about the sets $\mathcal{P}(\mathbb{N})$ and $(0, \infty)$?