

Homework
due on Friday, May 13

Read carefully Chapter 3 and the notes about logic linked on the course web page. Solve the following problems.

Problem 1. a) Complete the definition of the limit using quantifiers and the symbols from logic:

$$\lim_{n \rightarrow \infty} a_n = L \Leftrightarrow \dots$$

b) Use a) and the equivalence of the propositions $p \Leftrightarrow q$ and $\neg p \Leftrightarrow \neg q$ to complete the following proposition:

$$\lim_{n \rightarrow \infty} a_n \neq L \Leftrightarrow \dots$$

c) Use quantifiers and logic symbols to express the statement that $\lim_{n \rightarrow \infty} a_n$ does not exist (i.e. no L is the limit of (a_n)).

Problem 2. a) Prove that $\neg(p \Rightarrow q) \Leftrightarrow (p \wedge \neg q)$ is a tautology.

b) Prove that $[(p \Rightarrow q) \wedge \neg q] \Rightarrow \neg p$ is a tautology.

c) Prove that $(p \Rightarrow q) \Leftrightarrow (\neg q \Rightarrow \neg p)$ is a tautology.