Homework

due on Wednesday, October 3

Problem 1. Let $f : R \longrightarrow S$ be a surjective homomorphism of rings. Suppose that R is unital. Prove that S is also unital and $f(1)$ is the identity element of S.

Problem 2. a) Prove that $a \in \mathbb{Z}/n$ is invertible iff $\gcd(a, n) = 1$. What is the number of invertible elements of $\mathbb{Z}/n$?

b) Prove that $a \in \mathbb{Z}/n$ is a zero divisor iff $\gcd(a, n) > 1$.

c) For positive integers m, n define a function $f : \mathbb{Z}/n \longrightarrow \mathbb{Z}/m$ by $f(a) \equiv a \pmod{m}$ (i.e. $f(a)$ is the remainder upon division of a by m). Prove that f is a homomorphism iff $m|n$.

Read sections 2.1, 2.2 of Cameron's book and section 3.1 of Lauritzen's book..