**Homework**

due on Friday, October 5

Read section 3.1 of Lauritzen's book and section 2.2.2 of Cameron's book. Solve problems 8, 9 at the end of Chapter 3 of Lauritzen's book (pages 138-139) and the following problem.

**Problem 1.** Let $R$ be a ring.

a) Let $f : \mathbb{Z} \longrightarrow R$ be a homomorphism and let $r = f(1)$. Prove that $r^2 = r$.

b) Let $r \in R$ be such that $r^2 = r$ (such elements are called **idempotents**). Prove that there is unique homomorphism $f : \mathbb{Z} \longrightarrow R$ such that $f(1) = r$.

c) Suppose that $R$ is a domain and $r \in R$ is an idempotent. Prove that either $r = 0$ or $R$ is unital and $r = 1$. Conclude that if $R$ is a unital domain, there exist unique homomorphisms $f : \mathbb{Z} \longrightarrow R$ which is not identically 0.