Homework

due on Wednesday, October 10

Read section 2.2.3 of Cameron's book. Solve Problem 2.16 and the following problem

Problem 1. Let R be a unital ring.

a) Suppose that I, J, K are ideals of R such that both I, K and J, K are coprime. Prove that IJ and K are coprime. Conclude that $I \cap J$ and K are coprime.

b) Suppose that $I_1, I_2, ..., I_m$ are ideals of R which are pairwise coprime (i.e. $I_i + I_j = R$ for $i \neq j$). Prove that the map

$$f : R/(I_1 \cap I_2 \cap ... \cap I_m) \longrightarrow (R/I_1) \oplus (R/I_2) \oplus ... \oplus (R/I_m),\ f(r + (I_1 \cap I_2 \cap ... \cap I_m)) = (r + I_1, ..., r + I_m)$$

is an isomorphism (use the second problem from problem set 16 and induction). This result is often called Chinease Remainder Theorem for rings (can you see the Chinese remainder theorem as a special case of this result?)

c) Suppose that R is commutative (and unital). Let $I_1, I_2, ..., I_m$ be pairwise coprime ideals of R. Prove that $I_1 \cap I_2 \cap ... \cap I_m = I_1 I_2 ... I_m$.