Homework

due on Friday, October 12

Read section 2.2.3 of Cameron's book. Solve the following problems:

Problem 1. How many ideals does the ring $\mathbb{Z}/60$ have?

Problem 2. Let I be the principal ideal $(1+3i)\mathbb{Z}[i]$ of the ring of Gaussian integers $\mathbb{Z}[i]$.

a) Prove that $\mathbb{Z} \cap I = 10\mathbb{Z}$.

b) Prove that $\mathbb{Z} + I = \mathbb{Z}[i]$.

c) Prove that $\mathbb{Z}[i]/I$ is isomorphic to $\mathbb{Z}/10$.

Problem 3. Let F be a finite field.

a) Prove that there is unique prime number p such that F contains a subring isomorphic to the field $\mathbb{Z}/p$. **Hint:** There is unique non-zero homomorphism from $\mathbb{Z}$ to F).

b) Prove that a vector space V over $\mathbb{Z}/p$ is finite iff it is finite dimensional. Prove that the number of elements of V is a power of p. **Hint.** Consider a basis of V.

c) Explain how F can be considered as a vector space over $\mathbb{Z}/p$, where p is defined in a) and conclude that the number of elements of F is a power of p.