Homework

due on Tuesday, October 16

Read section 3.2 of Lauritzen's book. Solve the following problem:

Problem 1. Let $f : R \longrightarrow S$ be a homomorphism of commutative unital rings.

a) Prove that if P is a prime ideal of S then $f^{-1}(P)$ is a prime ideal of R.

b) Find an example when P is a maximal ideal of S but $f^{-1}(P)$ is not maximal in R.

c) Prove that if f is onto and Q is a prime ideal of R such that $\ker f \subseteq Q$ then $f(Q)$ is a prime ideal of S.

d) Suppose that f is surjective. Prove that if P is a maximal ideal of S then $f^{-1}(P)$ is maximal in R. Prove that if Q is a maximal ideal of R then $f(Q)$ is either S or it is a maximal ideal of S. Show by example that a similar statement for prime ideals is false.

e) Find all prime ideals of $\mathbb{Z}/36\mathbb{Z}$.

Problem 2. Let R be a commutative unital ring.

a) Prove that R is a domain iff $\{0\}$ is a prime ideal of R.

b) Prove that if P is a prime ideal and $r \in R$ is nilpotent then $r \in P$.

c) Prove that if R is finite then every prime ideal of R is maximal.