Homework due on Tuesday, November 6

Read sections 3.4 in Lauritzen's book and sections 2.4, 7.2.3, 7.2.4 in Cameron's book. Solve the following problems.

Problem 1. Let R be a UFD. For a non-zero polynomial $f \in R[x]$ define the content $\operatorname{cont}(f)$ of f as a greatest common divisor of the coefficients of f (note: this is only defined up to invertible element of R; in other words, the content is not a unique element of R but a class of associated elements). Prove that $\operatorname{cont}(fg) = \operatorname{cont}(f)\operatorname{cont}(g)$. Hint: Use Gauss Lemma.

Problem 2. Let R be a UFD with a field of fractions K. Suppose that $f \in R[x]$ is monic. Prove that if $g \in K[x]$ is monic and g|f in K[x] then $g \in R[x]$.

Problem 3. Let $K \subseteq L$ be fields. Suppose that $f, g \in K[x]$ and f|g in the ring L[x]. Prove that f|g in the ring K[x].