Homework

due on Friday, November 9

Read section 2.1, 2.2 in Lauritzen's book and section 3.1 in Cameron's book. Solve the following problems.

Problem 1. Let G be a group and let $H = \{g \in G : g^2 = e\}$.

a) Prove that if G is abelian then H is a subgroup of G.

b) Is H a subgroup when $G = D_8$ is the dihedral group of order 8?

Problem 2. Let G be the set of all bijections $f : \mathbb{Z} \longrightarrow \mathbb{Z}$ which preserve distance, i.e. such that $|f(i) - f(j)| = |i - j|$ for all integers i, j.

a) Show that G is a subgroup of $\mathrm{Sym}(\mathbb{Z})$. It is called the **infinite dihedral group** and it is often denoted by D_∞.

b) The group G contains elements T, S such that $T(a) = a + 1$ and $S(a) = -a$ for all integers a. Prove that $S * T = T^{-1} * S$. Show that the subgroup $< T >$ is infinite. What is $< S >$?

c) Show that if $F \in G$ and $F(0) = 0$ then either $F = 1$ (the identity) or $F = S$.

d) Show that every element of G is of the fo T^i or ST^i for some integer i (try to use similar argument to the one we used for dihedral group of order n).

e) Suppose that $T^5 S^7 T^3 = S^a T^b$. Find a and b.

Problem 3. In the group $GL_2(\mathbb{C})$ of all invertible 2×2 matrices with entries in complex numbers consider the matrices $I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$, $i = \begin{pmatrix} \sqrt{-1} & 0 \\ 0 & -\sqrt{-1} \end{pmatrix}$, $j = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$, $k = ij = \begin{pmatrix} 0 & \sqrt{-1} \\ \sqrt{-1} & 0 \end{pmatrix}$. Let Q_8 be the set $\{I, -I, i, -i, j, -j, k, -k\}$.

a) Show that Q_8 is a subgroup of $GL_2(\mathbb{C})$. Write the table of multiplication in Q_8. Q_8 is called the **quaternion group**.

b) List all subgroups of Q_8.