**Homework**

due on Monday, November 12

Read section 2.2 in Lauritzen's book and section 3.2.1 in Cameron's book. Solve the following problems.

**Problem 1.** Let $G$ be a group and let $H$, $K$ be two subgroups of $G$ such that $H \cup K$ is also a subgroup. Prove that either $H \subseteq K$ or $K \subseteq H$.

**Problem 2.** Let $G$ be a group. Define the **center** of $G$ as the subset $Z(G)$ of all elements which commute with every element of $G$, i.e.

$$Z(G) = \{g \in G : ag = ga \text{ for every } a \in G\}.$$

a) Prove that $Z(G)$ is a subgroup of $G$.

b) Find $Z(D_6)$, $Z(D_8)$, $Z(Q_8)$ and $Z(D_\infty)$.

c) What is $Z(D_{2n})$?

**Problem 3.** Let $G$ be a group and let $H$, $K$ be subgroups of $G$.

a) Show that $H \cap K$ is a subgroup of $G$.

b) Suppose that $h_1, h_2 \in H$. Prove that $h_1(H \cap K) = h_2(H \cap K)$ iff $h_1K = h_2K$. Conclude that $[H : H \cap K] \leq [G : K]$.

c) Let $L$ be a subgroup of $H$. Prove that $[G : L]$ is finite iff both $[G : H]$ and $[H : L]$ are finite and then $[G : L] = [G : H][H : L]$. Hint: Show that if $aH \neq bH$ then $aL \neq bL$ and that $H/L$ is a subset of $G/L$.

d) Suppose that $[G : H]$ and $[G : K]$ are finite. Prove that $[G : H \cap K] \leq [G : H][G : K]$ (so, in particular, $[G : H \cap K]$ is finite).