Homework

due on Tuesday, November 13

Read section 2.3 in Lauritzen's book and section 3.2.1, 3.2.2, 3.3.1, 3.3.2 in Cameron's book. Solve the following problems.

Problem 1. Let G be a group and let H, K be two subgroups of G. Define the set $HK = \{hk : h \in H, k \in K\}.$

a) Prove that if both H and K are normal then $H \cap K$ is also a normal subgroup of G.

b) Prove that if H is normal then $H \cap K$ is a normal subgroup of K.

c) Prove that if H is normal then HK = KH and HK is a subgroup of G.

d) Prove that if both H and K are normal then HK is a normal subgroup of G.

e) What is HK when $G = D_{16}$, $H = \{I, S\}$, $K = \{I, T^2, T^4, T^6\}$? Can you give geometric description of HK?

Problem 2. Let G be the group from Problem 2 of Homework 31. Let $H = \{T_{a,b} \in G : a \text{ is a rational number}\}$. Prove that H is a normal subgroup of G.