Homework

due on Wednesday, November 14

Read sections 2.4, 2.5, 2.6, 2.7 in Lauritzen's book and sections 3.2.1, 3.2.2, 3.2.3, 3.3.1, 3.3.2 in Cameron's book. Solve the following problems.

Problem 1. Let G be a group.

a) Prove that if $a \in G$ has finite order n then, for any integer k, the order of a^k is $n/\gcd(n,k)$.

b) Suppose that $a \in G$ has order m and $b \in G$ has order n. Suppose furthermore that $ab = ba$. Prove that the order of ab divides $mn/\gcd(m,n)$. Prove that both $m/\gcd(m,n)$ and $n/\gcd(m,n)$ divide the order of ab. Conclude that $mn/\gcd(m,n)^2$ divides the order of ab. In particular, if $\gcd(m,n) = 1$ then the order of ab is equal to mn.

c) Suppose that $f : G \longrightarrow H$ is a homomorphism. If $a \in G$ has finite order n then $f(a)$ also has a finite order k which divides n.

d) What are the orders of T, S and ST in D_{2n}?

Problem 2. Let G be a group such that $G/Z(G)$ is cyclic (here $Z(G)$ is the center of G). Prove that G is abelian.

Problem 3. Let H be a subgroup of a group G such that $[G : H] = 2$. Prove that H is a normal subgroup of G.

Problem 4. Prove that every subgroup of the quaternion group Q_8 is normal.