Homework
due on Friday, November 16

Read sections 2.4, 2.5, 2.6, 2.7, 2.8 in Lauritzen's book and sections 3.2.1, 3.2.2, 3.2.3, 3.3.1, 3.3.2, 3.3.3 in Cameron's book.
Solve the following problems (problem 1 is a translation to our notation of Problem 3.20 in Cameron's book).

Problem 1. a) Let G be a group. For any element $g \in G$, let c_g be the conjugation by g map defined by $c_g(x) = gcg^{-1}$. Show that c_g is an automorphism of G. (It is called the **inner automorphism** induced by the element g.)

b) Show that the set $\{c_g : g \in G\}$ is a subgroup of Aut$G$. (This subgroup is called the **inner automorphism group** of G, denoted Inn$G$.)

c) Show that the map $g \mapsto c_g$ is a homomorphism from G to Aut$G$, whose image is the inner automorphism group Inn$G$ and whose kernel is the center $Z(G)$. Deduce that Inn$G \cong G/Z(G)$.

d) Show that Inn$G$ is a normal subgroup of Aut$G$. (By definition, the factor group Aut$G$/Inn$G$ is the **outer automorphism group** of G, denoted Out$G$.)

Problem 2. Let G be a group and let H be a subgroup of G. Define the **normalizer** of H in G as $N_G(H) = \{a \in G : aH = Ha\}$.

a) Prove that $a \in N_G(H)$ iff $c_a(H) = H$, where c_a is the conjugation by a.

b) Prove that $N_G(H)$ is a subgroup of G and that $H \subseteq N_G(H)$ and the center $Z(G) \subseteq N_G(H)$.

c) Prove that H is a normal subgroup of $N_G(H)$. Conculde that $H \lhd G$ iff $N_G(H) = G$.

d) Let $G = D_8$, $H = \{I, S\}$. Find $N_G(H)$.

Problem 3. Let G be a finite group and let H be a subgroup of G. Let m be the smallest positive integer such that $a^m \in H$. Prove that $< a^m >= H\cap < a >$ and that m divides the order of a. **Hint:** Prove first that $\{k \in \mathbb{Z} : a^k \in H\}$ is a subgroup of $\mathbb{Z}$.

Problem 4. Let G be a group and let M, N be normal subgroups of G. Suppose that G/M and G/N are abelian. Prove that $G/(M \cap N)$ is also abelian.