Homework

due on Tuesday, November 20

Read sections 2.9 in Lauritzen's book and sections 3.1.2 Example 3 and 3.4.3 in Cameron's book.

Solve the following problems.

Problem 1. a) Find the product

$$\begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 4 & 1 & 3 & 2 & 5 \end{pmatrix}^{-1} \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 2 & 1 & 3 & 4 & 5 \end{pmatrix} \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 4 & 1 & 3 & 2 & 5 \end{pmatrix}$$

b) Find the cycle decomposition and order of

1. $\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\ 3 & 1 & 4 & 2 & 7 & 6 & 9 & 8 & 5 \end{pmatrix}$;
2. $\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 \\ 7 & 6 & 5 & 4 & 3 & 2 & 1 \end{pmatrix}$;
3. $\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 \\ 7 & 6 & 5 & 4 & 3 & 2 & 1 \end{pmatrix} \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 \\ 2 & 3 & 1 & 5 & 6 & 7 & 4 \end{pmatrix}$.

c) Express the following permutations as products of disjoint cycles and find their orders

1. $(1, 2, 3, 4, 5)(1, 2, 3, 4, 6)(1, 2, 3, 4, 7)$;
2. $(1, 2, 3)(3, 4, 5, 6)(1, 2, 3)^{-1}$;
3. $(1, 2, 3, 4, 5, 6)^3$.

Express each of these permutations as a product of transpositions.

d) Find a permutation $\rho \in S_n$ such that $\rho\tau\rho^{-1} = \sigma$ or prove that no such ρ exists, where

1. $\tau = (1, 3, 4)$, $\sigma = (2, 3, 5)$, $n = 5$;
2. $\tau = (1, 4)(2, 3, 5)$, $\sigma = (2, 4, 1)(5, 3)$, $n = 5$;

3. $\tau = (1,3,5,7)(2,4,6)$, $\sigma = (1,6)(4,5,7,3,2)$.

Problem 2. Let σ, τ be two transpositions in S_n, $n \geq 3$. Show that $\sigma\tau$ can be expressed as a product of $3-$cycles (not necessarily disjoint).