

## Homework

due on Monday, November 26

Read sections 2.9 in Lauritzen's book and sections 3.1.2 Example 3 and 3.4.3 in Cameron's book.

Solve the following problems.

**Problem 1.** a) Which of the following permutations are even?

1.  $\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\ 2 & 4 & 5 & 1 & 3 & 7 & 8 & 9 & 6 \end{pmatrix};$

2.  $(1, 2, 3, 4, 5, 6)(7, 8, 9);$

3.  $(1, 2)(1, 2, 3)(4, 5)(5, 6, 8)(1, 7, 9).$

b) Prove that a  $k$ -cycle is even iff  $k$  is odd.

c) In the even permutation

$$\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\ 3 & 1 & 2 & & & & 7 & 8 & 9 & 6 \end{pmatrix}$$

two entries are missing. Find the missing entries.

**Problem 2.** Prove that every element of  $A_n$ ,  $n \geq 3$  is a product of 3-cycles.

**Problem 3.** a) Prove that the center of  $S_n$  is trivial.

b) Find the number of conjugacy classes in  $S_6$ .

c) Prove that the centralizer of  $(1, 2, \dots, k)$  in  $S_n$  has  $k(n - k)!$  elements.

**Problem 4.** Let  $H$  be a subgroup of  $S_n$  which contains  $(1, 2)$  and  $(1, 2, 3, \dots, n)$ . Prove that  $H = S_n$ . Hint: Show that  $H$  contains all transpositions.

# Happy Thanksgiving!