Homework
due on Friday, November 30

Read sections 2.10 in Lauritzen's book and sections 7.1.1, 7.1.2 in Cameron's book.

Solve the following problems.

Problem 1. Suppose that a group G acts on a set X. For $g \in G$ define a function $l_g : X \longrightarrow X$ by $l_g(x) = g * x$.

a) Prove that l_g is a bijection (so it belongs to $\mathrm{Sym}(X)$).

b) Prove that the map $G \longrightarrow \mathrm{Sym}(X)$, $g \mapsto l_g$ is a homomorphism.

c) Suppose that $\psi : G \longrightarrow \mathrm{Sym}(X)$ is a homomorphism. Define a function $* : G \times X \longrightarrow X$ by $g * x = \psi(g)(x)$. Prove that $*$ is a group action of G on X.

d) Show that b) and c) define inverse of each other bijections between actions of G on X and homomorphisms $G \longrightarrow \mathrm{Sym}(X)$.

Problem 2. Let G be a group and H a subgroup of G. Let $X = G/H$ be the set of all left cosets of H in G. Define $* : G \times X \longrightarrow X$ by $g * (aH) = (ga)H$. Prove that $*$ is an action of G on X. Show that this action has only one orbit (such action is called **transitive**). According to Problem 1 this action corresponds to a homomorphism $G \longrightarrow \mathrm{Sym}(X)$. Prove that kernel of this homomorphism is the largest normal subgroup of G which is contained in H (i.e. that any normal subgroup of G which is contained in H is also conatined in this kernel).

Problem 3. Suppose that a group G acts on a set X. Let k be a positive integer, $k \leq |X|$. Let $P_k(X)$ be the set of all subsets of X which have k elements. For $g \in G$ and $A = \{x_1, x_2, ..., x_k\} \in P_k(X)$ define $g * A = \{g * x_1, g * x_2, ..., g * x_k\}$. Prove that this defines an action of G of $P_k(X)$.