Homework

due on Monday, September 10

Problem 1. a) Prove that any natural number n such that $n \equiv 3 \pmod 4$ has a prime divisor p such that $p \equiv 3 \pmod 4$ (hint: note that every odd prime number q either satisfies $q \equiv 1 \pmod 4$ or $q \equiv 3 \pmod 4$; what can you say about product of primes of the first type?)

b) Prove that $n! - 1 \equiv 3 \pmod 4$ for any $n > 3$.

c) Prove that every prime divisor of $n! - 1$ is bigger than n.

d) Conclude that there are infinitely many primes q such that $q \equiv 3 \pmod 4$.

Problem 2. Prove that every composite number n has a prime divisor not larger than $\sqrt{n}$.

Problem 3. The numbers n, $n+2$, $n+4$ are prime. What is n? Prove your answer. (Hint: consider these numbers modulo 3).

Read section 1.8 of Lauritzen's book.