Test 1, take-home
due on Tuesday, April 1

Problem 1. Find the minimal polynomial of $\sqrt[3]{2}+\sqrt[3]{4}$ over $\mathbb{Q}$. What are the other roots of this polynomial?

Problem 2. Let L/K be a finite extension of fields. Suppose that $f \in K[x]$ is irreducible over K and its degree is relatively prime to $[L:K]$. Prove that f is irreducible in $L[x]$.

Problem 3. Let $\Phi_n(x)$ be the $n-$th cyclotomic polynomial and let p be a prime number.

a) Prove that if $p|n$ then $\Phi_{pn}(x) = \Phi_n(x^p)$.

b) Prove that if $p \nmid n$ then $\Phi_n(x^p) = \Phi_n(x)\Phi_{np}(x)$.

Problem 4. Let L be a field and let p be a prime number. Suppose that F_1, F_2, K are subfields of L such that F_1/K and F_2/K are Galois and both $Gal(F_1/K)$ and $Gal(F_2/K)$ are p-groups. Prove that the Galois groups of F_1F_2/K and $F_1 \cap F_2/K$ are also p-groups.

Problem 5. a) Prove that the Galois group of the splitting field of x^3-3 over $\mathbb{Q}$ is isomorphic to the symmetric group S_3 hence is nonabelian.

b) Prove that $\mathbb{Q}(\sqrt[3]{3})$ is not a subfield of any cyclotomic field $\mathbb{Q}(\zeta_n)$.