Homework

due on Tuesday, February 19

Problem 1. Let E/F be an algebraic extension. Suppose that every polynomial in $F[x]$ splits into linear factors in $E[x]$. Prove that every polynomial in $E[x]$ splits into linear factors.

Problem 2. Let a be a root of $x^2 + x + 1 \in \mathbb{Q}[x]$. Prove that $\mathbb{Q}(a) = \mathbb{Q}(\sqrt{a})$.

Problem 3. Prove that a regular 9-gon can not be constructed with a compas and a ruler.

Problem 4. Let $f, g \in F[x]$ be irreducible polynomials. Suppose that E is an extension of F and $a, b \in E$ satisfy $f(a) = 0 = g(b)$. Prove that f is irreducible over $F(b)$ iff g is irreducible over $F(a)$.

Problem 5. Is there a complex number a such that a and $a + 1$ have the same minimal polynomial over $\mathbb{Q}$?

Read carefully sections 13.3 and 13.4 from Dummit and Foote.