Homework 1

due on Wednesday, January 24

Read carefully Appendix A and sections 1.1, 1.2 in the book. Read the section about induction in the link on the course web page.. Solve the following problems.

Problem 1. Suppose that $a_1 = 2$ and $a_{n+1} = 3a_n + 2$. Prove that $a_n = 3^n - 1$ for every natural number n.

Problem 2. Prove by induction that every natural number is a sum of disctinct powers of 2 (e.g. $1 = 2^0$; $2 = 2^1$, $3 = 2^0 + 2^1$, etc.). Hint: In the inductive step consider the case when your number is even and the case when it is odd.

Extra credit: prove that such expression is unique. Hint: Observe that $1 + 2 + 4 + .. + 2^n < 2^{n+1}$

Problem 3. We defined in class $v(n)$ to be the number of positive divisors of n. Charcterize positive integrs n such that $v(n) = 3$.