Homework 2

due on Friday, January 26

Read sections 1.3, 1.4, 1.5 in the book. Solve problem 57 in Chapter 1 and the following problems.

Problem 1. Prove that if a, b are relatively prime integers such that $a|c$ and $b|c$ then $ab|c$. Hint: Write $c = ac_1$, $ua + wb = 1$ for some integers u, w and use this to show that $b|c_1$.

Problem 2. For positive integers a, b define $[a, b] = ab/\gcd(a, b)$.

a) Prove that $a/\gcd(a, b)$ and $b/\gcd(a, b)$ are relatively prime.

b) Prove that if $a|c$ and $b|c$ then $[a, b]|c$.

c) Conlcude that $[a, b]$ is the smallest positive integer divisible by both a and b (we call it the **least common multiple of** a **and** b).

Problem 3. Let $F_n = 2^{2^n} + 1$, for $n = 0, 1, 2, ...$

a) Prove that $F_0 \cdot F_1 \cdot F_2 \cdot ... \cdot F_n = F_{n+1} - 2$ for every n.

b) Prove that $\gcd(F_n, F_m) = 1$ for $n \neq m$.

c) Use b) to give yet another proof that the set of primes is infinite.