Homework

due on Wednesday, December 1

Read carefully Chapter 7, sections 37, 39 of Hartshorne's book. Solve the following problems:

Problem 1. Consider 4 circles S_1, S_2, S_3, S_4. Suppose that S_1, S_2 intersect at A_1 and A_2, circles S_2, S_2 intersect at B_1 and B_2, circles S_3, S_4 intersect at C_1 and C_2, and circles S_4, S_1 intersect at D_1 and D_2. Suppose furthermore that the points A_1, B_1, C_1, D_1 are on a circle S (or on a line). Prove that the points A_2, B_2, C_2, D_2 are also on a circle (or on a line).

Hint: Consider a circular inversion with center A_1 (in any circle with center A_1). What will hapen to S_1, S_2 and S? What will happen ntto the other circles? You should be able to apply our old theorem about Miquel point.

Problem 2. No three of the points A, B, C, D are collinear. Prove that the angle between the circumcircles of triangles ABC and ABD is the same as the angle between the circumcircles of triangles ACD and BCD.

Hint: Perform a circular inversion in a circle with center A. What happens to the angles in question?

Problem 39.19

Problem 39.10

Problem 39.11

Hint: We can find the Euclidean center C of ζ (how ?). Consider the line OC. Explain why the P-center of ζ is on OC. Now we may proceed in various ways.

First approach: It suffices to construct a circle T which is perpendicular to both Γ and ζ. Explain why T will give a P-line which contains a P-diameter of ζ (note: in any Hilbert plane a tangent at a point is perpendicular to the diameter from that point). Thus the point inside ζ where T intersects the line OC is the P-center of ζ. To construct T, note first that it is easy to construct a circle perpendicular to a given line and a given circle. Then use appropriate inversion to reduce to this case.

Second approach: Use first appropriate inversion to reduce to the case when ζ passes through O. Assuming that ζ contains O, the idea from the first approach becomes even simpler. Let A be the second point of intersection of the line OC and ζ. Find the inverse B of A in the circle Γ. Then B is outside of Γ. Draw a tangent to Γ from B and let D be the point of tangency. Prove that the circle with center at B and radius BD intersects the segment OA at the P-center of ζ.

Regardless of which approach you use, carefully explain how you do the appropriate inversions.