Homework
due on Monday, October 15

Read carefully Chapter 1 of Hartshorne's book. Read Books 1-4 of the "Elements". Solve the following problems in Chapter 1:

Problem 2.24. Hint: Let O_1, O_2 be the centers of the circles, r_1, r_2 corresponding radii with $r_2 \geq r_1$. Prove that solution exists iff $O_1O_2 \geq r_2 - r_1$. Explain why it suffices to construct a right angled traingle O_1O_2D with hypotenuse O_1O_2 and $DO_2 = r_2 - r_1$. If $O_1O_2 \geq r_1 + r_2$ there is also a second solution. Explain how to construct it.

Problem 3.8. Hint: Consider the circle through A, D, B.

Problems 3.10, 3.11.

Problem 4.7.

Problem 5.5.

Your solutions should only use the results from books 1-4 of the Elements and Chapter 1 of the book, and you should carefully explain what propositions you are using.

Also solve the following problems.

Problem 1. In class we proved the following theorem.

Theorem. Let ABC be a triangle and let P be a point whose orthogonal projections on the sides of the triangle are K, L, M. Then the points K, L, M are collinear if and only if P is on the circumscribed circle of the triangle ABC. The line through K, L, M is then called **the Simson line of** P.

Using this result solve the following problem.

Let A, B, C be three collinear points and let P be a point outside the line through A, B, C. Prove that the circumcenters of the triangles PAB, PAC, PBC and the point P lie on a circle. Hint: Note that if two circles intesect at two points X, Y then the line joining the centers of the circles is the perpendicular bisector of XY. Consider the triangle with vertices at the circumcenters. What are the projections of P on the sides of this triangle?

Problem 2. Let ABC be a triangle and let a line l intersect the lines AB, BC, AC

at C_1, A_1, B_1 respectively. Let P be the corresponding Miquel point. Prove that the orthogonal projections of P on the lines AB, AC, BC and l are collinear.

Problem 3. The altitudes of a triangle ABC intersect at a point H. Let O_A be the circumcenter of the triangle BCH. Similarly define O_B and O_C. Prove that the segments AO_A, BO_B, CO_C share a common midpoint. What is this point? Conclude that the triangles ABC and $O_AO_BO_C$ are congruent.

Hint: 1) What is the orthocenter of BCH?

2) What can you say about the nine-point circles of the triangles ABC, ABH, BCH, ACH?

b) Prove that H is the circumcenter of $O_AO_BO_C$ and that the circumcenter of ABC is the orthocenter of $O_AO_BO_C$.

c) Prove that the Euler lines of the triangles ABC, ABH, BCH, ACH intersect at one point.