

Homework

due on Wednesday, November 28

Read carefully Chapter 7, sections 37, 39 of Hartshorne's book. Solve the following problems:

Problem 1. Consider 4 circles S_1, S_2, S_3, S_4 . Suppose that S_1, S_2 intersect at A_1 and A_2 , circles S_2, S_3 intersect at B_1 and B_2 , circles S_3, S_4 intersect at C_1 and C_2 , and circles S_4, S_1 intersect at D_1 and D_2 . Suppose furthermore that the points A_1, B_1, C_1, D_1 are on a circle S (or on a line). Prove that the points A_2, B_2, C_2, D_2 are also on a circle (or on a line).

Hint: Consider a circular inversion with center A_1 (in any circle with center A_1). What will happen to S_1, S_2 and S ? What will happen to the other circles? You should be able to apply our old theorem about Miquel point.

Problem 2. No three of the points A, B, C, D are collinear. Prove that the angle between the circumcircles of triangles ABC and ABD is the same as the angle between the circumcircles of triangles ACD and BCD .

Hint: Perform a circular inversion in a circle with center A . What happens to the angles in question?

Problem 39.19

Problem 39.10

Problem 39.11

Hint: We can find the Euclidean center C of ζ (how?). Consider the line OC . Explain why the P -center of ζ is on OC . Now we may proceed in various ways.

First approach: It suffices to construct a circle T which is perpendicular to both Γ and ζ . Explain why T will give a P -line which contains a P -diameter of ζ (note: in any Hilbert plane a tangent at a point is perpendicular to the diameter from that point). Thus the point inside ζ where T intersects the line OC is the P -center of ζ . To construct T , note first that it is easy to construct a circle perpendicular to a given line and a given circle. Then use appropriate inversion to reduce to this case.

Second approach: Use first appropriate inversion to reduce to the case when ζ passes through O . Assuming that ζ contains O , the idea from the first approach

becomes even simpler. Let A be the second point of intersection of the line OC and ζ . Find the inverse B of A in the circle Γ . Then B is outside of Γ . Draw a tangent to Γ from B and let D be the point of tangency. Prove that the circle with center at B and radius BD intersects the segment OA at the P -center of ζ .

Regardless of which approach you use, carefully explain how you do the appropriate inversions.