Homework

due on Wednesday, September 25

Read carefully Chapter 1 of Hartshorne's book. Study Books 1,2,3,4 of the "Elements". Solve problems 1.10, 2.12, 2.14, 2.16 in Chapter 1. Work on several other problems in this chapter.

When you do a construction, write carefully all steps of the construction as discussed in the book. Then provide a proof that your construction works. When you claim that some lines intersect, explain why. Include carefully done drawing using a compas and a ruler.

Problem 1. Let ABC be a triangle. Let t_A and t_B be perpendicular bisectors of BC and AC respectively. Use the 5th postulate and the results of book 1 to show that t_A and t_B intersect.

Problem 2. Suppose that the lines AB and CD intersect at a point P such that $PA \cdot PB = PC \cdot PD$ and such that $A$is between points P and B and C is between points P and D. Prove that points A, B, C, D are on one circle. You can use results from books 1-4 of the Elements.

Problem 3. Let ABC be an isosceles traingle ($AB = AC$). Suppose that M is a point on the side AB such that $AB \cdot MB = AM^2$. Suppose also that $AM = BC$. Prove that the angle $\angle ABC$ is twice the angle $\angle BAC$.

Hint. Consider the cirumcircle Ω of traingle AMC. Show that BC is tangent to Ω. Use this to show that $\angle CMB = \angle CBM$. In your proof you can use results from books 1-3 of the Elements.