Homework
due on Wednesday, September 25

Read carefully Chapter 1 of Hartshorne’s book. Study Books 1,2,3,4 of the "Elements". Solve problems 1.10, 2.12, 2.14, 2.16 in Chapter 1. Work on several other problems in this chapter.

When you do a construction, write carefully all steps of the construction as discussed in the book. Then provide a proof that your construction works. When you claim that some lines intersect, explain why. Include carefully done drawing using a compas and a ruler.

Problem 1. Let $ABC$ be a triangle. Let $t_A$ and $t_B$ be perpendicular bisectors of $BC$ and $AC$ respectively. Use the 5th postulate and the results of book 1 to show that $t_A$ and $t_B$ intersect.

Problem 2. Suppose that the lines $AB$ and $CD$ intersect at a point $P$ such that $PA \cdot PB = PC \cdot PD$ and such that $A$ is between points $P$ and $B$ and $C$ is between points $P$ and $D$. Prove that points $A, B, C, D$ are on one circle. You can use results from books 1-4 of the Elements.

Problem 3. Let $ABC$ be an isosceles triangle ($AB = AC$). Suppose that $M$ is a point on the side $AB$ such that $AB \cdot MB = AM^2$. Suppose also that $AM = BC$. Prove that the angle $\angle ABC$ is twice the angle $\angle BAC$.

Hint. Consider the cirumcircle $\Omega$ of triangle $AMC$. Show that $BC$ is tangent to $\Omega$. Use this to show that $\angle CMB = \angle CBM$. In your proof you can use results from books 1-3 of the Elements.