Homework

due on Wednesday, October 16

Read carefully Chapter 1 of Hartshorne's book. Read Books 1-4 of the "Elements". Study the links on the course web-page. Solve the following problems in Chapter 1:

Problem 2.24. Hint: Let O_1, O_2 be the centers of the circles, r_1, r_2 corresponding radii with $r_2 \geq r_1$. Prove that solution exists iff $O_1O_2 \geq r_2 - r_1$. Explain why it suffices to construct a right angled traingle O_1O_2D with hypotenuse O_1O_2 and $DO_2 = r_2 - r_1$. If $O_1O_2 \geq r_1 + r_2$ there is also a second solution. Explain how to construct it.

Problem 3.8. Hint: Consider the circle through A, D, B.

Problems 3.10, 3.11.

Problem 4.7.

Problem 5.5.

Your solutions should only use the results from books 1-4 of the Elements and Chapter 1 of the book, and you should carefully explain what propositions you are using. Work on several other problems to section 5.

Also solve the following problems.

Problem 1. In class we proved the following theorem.

Theorem. Let ABC be a triangle and let P be a point whose orthogonal projections on the sides of the triangle are K, L, M. Then the points K, L, M are collinear if and only if P is on the circumscribed circle of the triangle ABC. The line through K, L, M is then called **the Simson line of** P.

Using this result solve the following problem.

Let A, B, C be three collinear points and let P be a point outside the line through A, B, C. Prove that the circumcenters of the triangles PAB, PAC, PBC and the point P lie on a circle. Hint: Note that if two circles intesect at two points X, Y then the line joining the centers of the circles is the perpendicular bisector of XY. Consider the triangle with vertices at the circumcenters. What are the projections of P on the sides of this triangle?

Problem 2. Let ABC be a triangle and let a line l intersect the lines AB, BC, AC at C_1, A_1, B_1 respectively. Let P be the corresponding Miquel point. Prove that the orthogonal projections of P on the lines AB, AC, BC and l are collinear.

Problem 3. The altitudes of a triangle ABC intersect at a point H. Let O_A be the circumcenter of the triangle BCH. Similarly define O_B and O_C. Prove that the segments AO_A, BO_B, CO_C share a common midpoint. What is this point? Conclude that the triangles ABC and $O_AO_BO_C$ are congruent.

Hint: 1) What is the orthocenter of BCH?

2) What can you say about the nine-point circles of the triangles ABC, ABH, BCH, ACH?

b) Prove that H is the circumcenter of $O_AO_BO_C$ and that the circumcenter of ABC is the orthocenter of $O_AO_BO_C$.

c) Prove that the Euler lines of the triangles ABC, ABH, BCH, ACH intersect at one point.