Homework

due on Wedesday, November 13

Study Chapters 3 and 4 of Hartshorne's book. Solve the following problems:

15.3

18.9

20.4

20.5

20.14 Hint: Let the given lines be A_1A_2, A_2A_3, A_1A_3 and l, where l intesects A_1A_2 at B_3, A_1A_3 at B_2 and A_2A_3 at B_1. Consider cirles C_1 on diameter A_1B_1 and C_2 on diameter A_2B_2. Prove that each of the four orthocenters has the same power with respect to C_1 and C_2.

Problem 1. In any Hilbert plane, prove that the interior of a circle is convex. Hint: Prove first that if ABC is a triangle and X is between A and B then either $\overline{AX} < \overline{AB}$ or $\overline{AX} < \overline{AC}$. You may use propositions 2-27 from book 1 of Euclid.

Problem 2. Let Π be a Hilbert plane which does not satisfy the Archimedes axiom (A). Thus there exist segments $\overline{AB}$ and $\overline{AQ}$ such that $n\overline{AB} < \overline{AQ}$ for every natural number n. Consider the set Π_A which consists of A and all points P in Π for which there exists a natural number m such that $\overline{AP} < m\overline{AB}$ (we call such points finitely bounded from A). Call a subset l of Π_A a line if it is non-empty and there is a line L in Π such that $l = L \cap \Pi_A$.

a) Prove that Π_A with the lines defined above is an incidence geometry.

b) Prove that Π_A does not satisfy the parallel postulate (P) (hint: note first that for any $P \in \Pi_A$ the line PQ in Π intersected with Π_A is a line in Π_A through P and all these lines are parallel to each other.

c) Define betweenness in Π_A as follows: Y is between X and Z in Π_A if the same holds when we consider them as points in Π. Prove thet the betweenness axioms are satisfied for Π_A.

d) Define two segments $\overline{XY}$ and $\overline{KL}$ in Π_A to be congruent if the segments $\overline{XY}$ and $\overline{KL}$ in Π are congruent. Similarly, two angles $\angle XYZ$ and $\angle KLM$ are congruent in Π_A if the angles $\angle XYZ$ and $\angle KLM$ are congruent in Π. Prove that Π_A satisfies the congruence axioms. It follows that Π_A is a Hilbert plane which does not satisfy the parallel axiom.